Semiconductor Surface Emitting Lasers For Entangled Photons Generation

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We study the feasibility of using a resonant Vertical-Cavity Surface-Emitting Lasers (VCSELs) for the efficient generation of entangled photons. By taking advantage of the relatively high value of the third order non-linearity of the VCSEL, we focus on degenerate four-wave-mixing in the spontaneous regime. We calculate the two-photon production rate, the spectrum of the generated signal photons and the signal-idler cross-correlations. We determine how the production rate of the entangled photons is affected by the dispersion of the third order non-linear medium. Based on our results we identify the characteristics of a VCSEL that would be suitable for entangled photons generation.

Introduction

Modern quantum technology requires single and entangled photon sources suitable for on-chip integration [1, 2]. Considering their current size and manufacturing process, VCSEL devices appear to be easily scalable [3, 4, 5]. Cost-effectiveness and ubiquity of VCSEL lasers [6, 7] are the main drivers for this analysis. Massive arrays of VCSEL sources generating entangled photons would result into a number of applications in modern quantum technologies. Furthermore, the mature VCSEL technology gives the advantage of developing monolithic, electrically injected entangled photon source. Degenerate four wave mixing allows the generation of entangled photons whose frequencies are equidistant from the pump frequency, potentially allowing easy separation of the generated photons by filtering. The efficient conversion of two pump photons into a signal and an idler photons by degenerate four wave mixing requires that the cavity and the non-linear material allow both energy ($2\omega_{pump} = \omega_{signal} + \omega_{idler}$) and momentum ($2k_{pump} = k_{signal} + k_{idler}$) conservations. The latter is also referred to as phase matching. Highly dispersive material hinder such conservation [8, 9] and we take into account the impact of the spectral walk-off in our study. To evaluate the feasibility of generating entangled photons by degenerate four wave mixing from a VCSEL, we estimate the two-photon production rate, the frequency spectrum of the generated photons and their cross-correlation. We neglect other physical effects that might "disturb" the two-photon production, such as Raman scattering, Brillouin scattering or thermo-optical oscillations.

Two-Photon Wave Function

The resonant VCSEL cavity is assumed to be pumped by a classical, undepleted strong pump field:

$$E_p^{(+)}(x,t) = E_p e^{i\left[k_p(\omega_p)x - \omega_p t\right]}. \tag{1}$$

Let m_p, m_s and m_i denote respectively the wave numbers of the resonant pump, signal and idler modes. The corresponding signal and idler angular frequencies are denoted by ω_{m_s} and ω_{m_i}. We also define $\Delta m_s = m_s - m_p$ and $\Delta m_i = m_p - m_i$. The signal and idler fields $E^{(+)}_{s/i}(x,t)$ travelling in the positive x direction in the lossy VCSEL cavity are:

$$E^{(+)}_{s/i}(x,t) = \frac{1}{2}\sum_{\Delta m_{s/i}=1}^{\infty}\left(\frac{\hbar\omega_{m_{s/i}}}{2\pi\varepsilon_0 V n(\omega_{m_{s/i}})^2}\right)^{1/2} \int_{-\infty}^{+\infty} d\Omega a(\omega_{m_{s/i}}+\Omega)\frac{\sqrt{\gamma_{s/i}}}{\frac{\gamma_{s/i}}{2}+i\Omega} e^{i\left[k_{s/i}(\Omega)x-(\omega_{m_{s/i}}+\Omega)t\right]}, \quad (2)$$

where $k_{s/i}(\Omega) = \left(\omega_{m_{s/i}}+\Omega\right) n(\omega_{m_{s/i}}+\Omega)/c$ [10], $\gamma_{s/i}$ is the damping rate of the signal/idler wave in the one-side cavity [8], $n(\omega)$ is the refractive index of the resonant medium and V is the volume of the resonant VCSEL cavity. The interaction Hamiltonian describing the degenerate four wave mixing has the following expression:

$$H_{int} = \frac{3}{2}\varepsilon_0 \int_V \chi^{(3)} E_p^{(+)} E_p^{(+)} E_s^{(-)} E_i^{(-)} dV + H.c.\,, \quad (3)$$

where $\chi^{(3)}$ is the third-order non-linear susceptibility [8] and the signal and idler modes are such that $m_s + m_i = 2m_p$, that is, $\Delta m_s = \Delta m_i \equiv \Delta m$. The off-diagonal terms of the third order non-linear susceptibility tensor have been neglected and $\chi^{(3)}$ is considered as a constant parameter below. We calculate the state of the radiation field in the VCSEL cavity by means of first order perturbation theory:

$$|\psi_{cavity}\rangle \approx |0\rangle + \frac{1}{i\hbar}\int_0^{\Delta t} H_{int}(t)\,dt\,|0\rangle = |0\rangle + |\psi\rangle,$$

which defines the (non-normalized) perturbation state $|\psi\rangle$ that is referred to as the two-photon state. Note that the cavity can be assumed to be in the vacuum state before each two-photon emission if the production rate is significantly lower than the cavity damping rates: $\langle\psi|\psi\rangle/\Delta t \ll \gamma_{s/i}$ [10, 11]. As we shall see, it is verified in our case. Inserting the expressions of the fields in the interaction Hamiltonian (3), we obtain, for the two-photon state:

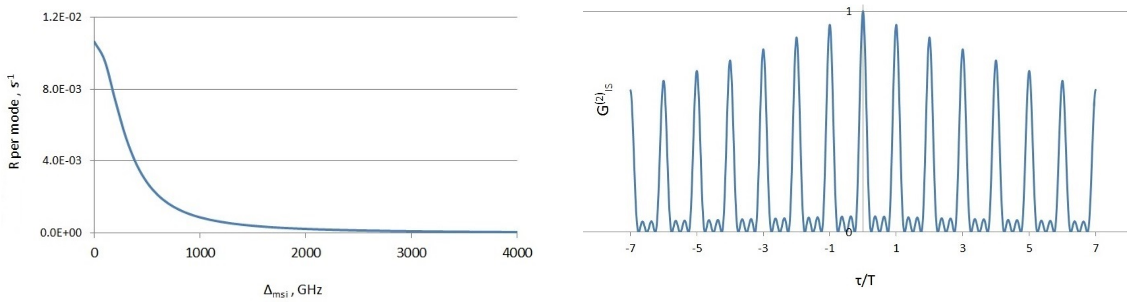
$$\begin{aligned}|\psi\rangle &= \frac{3}{16\pi}\chi^{(3)} E_p^2 \Delta t\,(\gamma_s\gamma_i)^{1/2}\sum_{\Delta m=1}^{\infty}\frac{(\omega_{m_s}\omega_{m_i})^{1/2}}{n(\omega_{m_s})n(\omega_{m_i})}\int d\Omega\int d\Omega' \frac{S_{si}(\Omega,\Omega')}{\left(\frac{\gamma_s}{2}-i\Omega\right)\left(\frac{\gamma_i}{2}-i\Omega'\right)} \\ &\times\; e^{i(\Omega+\Omega'-\Delta_{msi})\frac{\Delta t}{2}}\,\mathrm{sinc}\left[(\Omega+\Omega'-\Delta_{msi})\frac{\Delta t}{2}\right] a^{\dagger}(\omega_{m_s}+\Omega)a^{\dagger}(\omega_{m_i}+\Omega')\,|0\rangle, \quad (4)\end{aligned}$$

where we have defined the mode-dependent spectral walk-off $\Delta_{msi} = 2\omega_p - \omega_{m_s} - \omega_{m_i}$ and the function $S_{si}(\Omega,\Omega') = e^{i\left(2k_p(\omega_p)-k_s(\Omega)-k_i(\Omega')\right)\frac{L}{2}}\,\mathrm{sinc}\left[(2k_p(\omega_p)-k_s(\Omega)-k_i(\Omega'))\frac{L}{2}\right]$.

Two-Photon Production Rate and Output Spectrum

To compute the two-photon production rate $R = \frac{\langle\psi|\psi\rangle}{\Delta t}$, we perform a calculation similar to Ref. [10]. Starting from (4), we obtain:

$$R = \frac{1}{2\pi}\left(\frac{3}{8}\chi^{(3)}E_p^2\right)^2 \sum_{\Delta m=1}^{\infty}\frac{\gamma_s\gamma_i\omega_{m_s}\omega_{m_i}}{n^2(\omega_{m_s})n^2(\omega_{m_i})}\int\frac{d\Omega\,|S_{si}(\Omega,\Delta_{msi}-\Omega)|^2}{\left[\left(\frac{\gamma_s}{2}\right)^2+\Omega^2\right]\left[\left(\frac{\gamma_i}{2}\right)^2+(\Delta_{msi}-\Omega)^2\right]}. \quad (5)$$

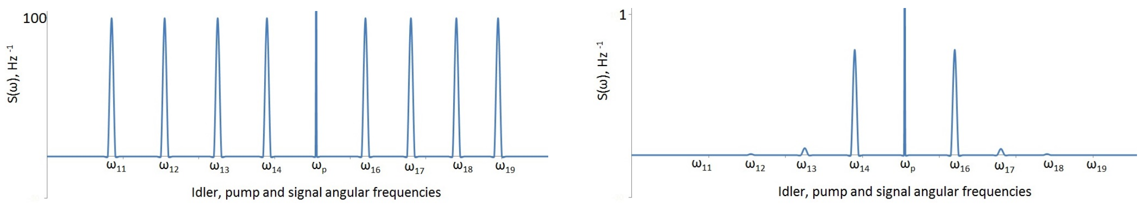


(a) Production rate

(b) Cross-correlations

Figure 1: (a) The production rate R per resonant mode is calculated according to (5) with the parameters: $\gamma_i = \gamma_s = 300\text{GHz}$, $\chi^{(3)} = 1.4 \times 10^{-18}\text{m}^2\text{V}^{-2}$, $\omega_p = 2.2 \times 10^{15}\,\text{s}^{-1}$, $E_p = 2 \times 10^4\text{Vm}^{-1}$ and $n_p = 3.4$. Assumption $\frac{\omega_{m_s}\omega_{m_i}}{n^2(\omega_{m_s})n^2(\omega_{m_i})} \approx \frac{\omega^2{}_p}{n_p^4}$.
(b) Normalized signal-idler cross-correlation function $G_{IS}^{(2)}(\tau)$ for $\gamma = 30\text{GHz}$, free spectral range $\Delta\omega = 3\text{THz}$ and $\Delta m = 1,..,4$. The horizontal scale is τ/T where $T = 2\pi/\Delta\omega$.

With $\Delta m_s = \Delta m_i = \Delta m$, we can safely approximate the $|S_{si}(\Omega, \Delta_{msi} - \Omega)|^2$ factor by 1. The two-photon production rate is depicted in Figure 1(a) which shows that the production rate drops dramatically when the spectral walk-off increases, in particular if the spectral walk-off has a value larger than the cavity damping rate γ_i ($\sim$ 300GHz in our case). With the values used in Figure 1 and $\Delta_{msi} = 3.4 \times 10^{12}$ (value applicable to the VCSEL we have chosen), we obtain a two-photon production rate when there is no compensation of the spectral walk-off: $R \approx 8.4 \times 10^{-5}\,\text{s}^{-1}$. The output spectrum $S(\omega)$ is a frequency comb (see Figure 2), as already described in Refs. [9] and [8]. For each signal and idler resonant frequency, the shape of the spectrum is given by the integrand in Eq. (5). The



(a) No spectral walk-off

(b) With spectral walk-off

Figure 2: Normalized signal-idler output spectrum for $m_s = 16,..,19$ and $m_i = 11,..,15$.

number of "teeth" of the comb is limited by the absorption band of the VCSEL so that our comb has only four coupled "teeth" at each side of the pump frequency, significantly less than in other cases [9]. If the spectral walk-off is not compensated in the VCSEL, the "amplitude" of the "teeth" decreases significantly with increasing Δm (that is, when the "teeth" get further away from the pump frequency) [8].

Signal-Idler Cross-Correlation

The signal-idler cross-correlation function is defined [10] as

$$G_{IS}^{(2)}(\tau) = \langle\psi|\, E_{Out_i}^{(-)}(x,t)\, E_{Out_s}^{(-)}(x,t+\tau)\, E_{Out_s}^{(+)}(x,t+\tau)\, E_{Out_i}^{(+)}(x,t)\,|\psi\rangle, \tag{6}$$

and has a shape similar to the one obtained for an optical parametric oscillator in Ref. [11]. The function $G_{IS}^{(2)}(\tau)$ is depicted in Figure 1(b).

Conclusion

The calculation of the production rate has highlighted the significant impact of the dispersion when the spectral walk-off is large compared to the cavity linewidth γ [9]. Despite the relatively low value of the two-photon production rate that we could obtain so far (we have used the parameters of commonly available VCSELs), we find it worth investigating degenerate four wave mixing in a VCSEL as a potential source for entangled photons. The key challenge is to increase the production rate and for that purpose, the VCSEL cavity should be designed (i) to be resonant for the pump and at least one signal and idler modes and - whenever possible - several signal and idler couples of modes (ii) with a high value of the third order non-linearity $\chi^{(3)}$ (iii) with a compensation of the spectral walk-off to the maximum possible extent (iiii) to avoid undesirable (in our case) effects of Raman or Brillouin scattering as well as thermo-optical oscillations - in particular if the pump is stepped up in order to increase the production rate.

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