Efficient frequency conversion via Floquet modes in time-modulated cavities

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Time dependency of photonic structures has been an interesting topic for the past few years, leading to new physics and applications. Here we present a mechanism to achieve frequency conversion using a system of two time-modulated cavities. This setup allows for a precise optimization of the conversion efficiency by tuning the system parameters. We show that the dynamic modes of the coupled system (Floquet modes) and their band gaps play an important role in the process.

Introduction

In this work we aim to use a time dependent system of two coupled cavities to achieve tunable frequency conversion. This is a very general system, and it could be implemented e.g. in the context of plasmonic resonances (graphene example in Fig. 1 right), as these are strongly dependent on the material parameters.

Coupled Mode Theory (CMT)

We describe the two-cavity setup by a system of coupled equations (Fig. 1 left). We consider the case where the energy is injected directly into one of the modes. Moreover, one mode is dark while the other is bright; this allows to better quantify the conversion efficiency of the system, since we are interested in converting light to a different frequency. Since the time modulation relies on the cavity resonance frequency being dependent on time, the usual resonance frequencies become functions of time. The CMT equations for a system of two coupled cavities are:

$$\begin{cases} \dfrac{da(t)}{dt} = -i\omega_1(t)a(t) + \kappa b(t) + s(t) \\ \dfrac{db(t)}{dt} = -i\omega_2(t)b(t) + \kappa a(t) - \gamma b(t) \end{cases}$$

where $a(t)$ and $b(t)$ are respectively the dark and bright mode amplitudes, γ is the outcoupling coefficient, κ is the coupling coefficient between the two cavities, $\omega_{1,2}(t)$ are the time modulated resonance frequencies, and $s(t)$ is the source [1] (Fig. 1 left). We consider a periodic time modulation:

$$\omega_{1,2}(t) = \omega_{1,2} + \delta \sin \Omega t$$

where $\omega_{1,2}$ is the 'static' resonance frequency, δ is the modulation amplitude and Ω is the modulation frequency. In the previously studied geometry with a single cavity, the output is a frequency comb where the frequency components are separated by the modulation frequency Ω [2,3]. With two cavities, it is possible to tailor the comb, and to enhance some frequencies selectively for example, as we will show.

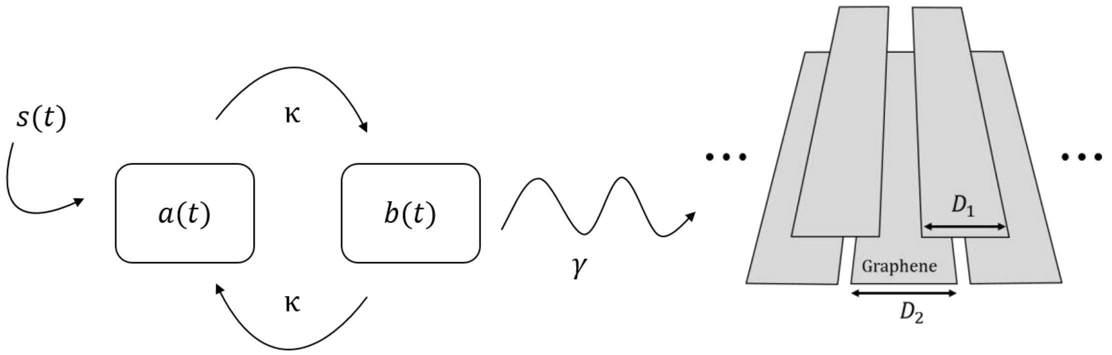

Figure 1: Representation of the modulated cavity system (left). Example of a possible implementation using two coupled graphene plasmonic gratings (right).

Floquet Modes

For a static system (i.e. $\omega_{1,2}(t) = \omega_{1,2}$), the system has 2 modes with frequencies $\omega_{\pm} = \frac{\omega_1+\omega_2}{2} \pm \frac{1}{2}\sqrt{(\omega_1 - \omega_2)^2 + 4\kappa^2}$. In the case where $\omega_1 = \omega_2$, the two mode frequencies take the simple form $\omega_{\pm} = \omega_{1,2} \pm \kappa$. With the introduction of a time modulation, new modes are introduced, which are called ‘Floquet modes’, and which exhibit harmonics for every Ω. The crossing of these new modes opens a bandgap for some specific κ values (Figure 2). To compute the Floquet mode frequencies, the trick is to consider the coupled mode equations in the frequency domain and to suppose that the solutions $a(t)$ and $b(t)$ have the form $a(t) = A(t)\exp(-i\omega_+ t)$ and $b(t) = B(t)\exp(-i\omega_- t)$ where $A(t)$ and $B(t)$ have a periodicity Ω. Injecting the Fourier series of these solutions into the frequency versions of the coupled mode equations leads to an eigenvalue equation [4]. These eigenvalues are the Floquet mode frequencies, represented in Figure 2.

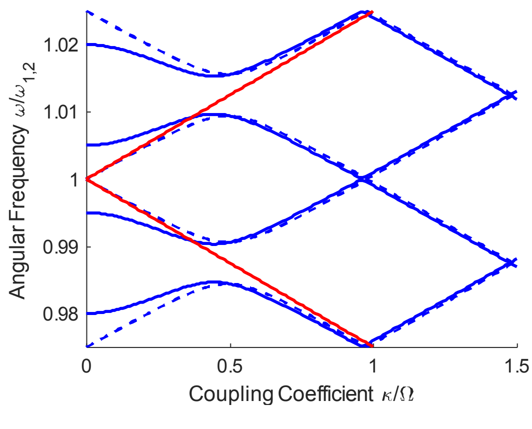

Figure 2: Band-like structure of the Floquet mode frequencies. Solid lines indicate the non-degenerate case ($\omega_1 \neq \omega_2$) and dashed lines the degenerate case ($\omega_1 = \omega_2$). The red lines are the ‘static modes’ (for the degenerate case). Gaps open when branches cross at $\kappa = \Omega/2$.

The modulation amplitude δ is the parameter responsible for the bandgap size: the higher δ the larger the bandgap. Another advantage of this bandgap is that the frequency conversion is most efficient when the excitation frequency ω_0 is equal to a Floquet mode frequency at the edge of the bandgap. In order to characterize the conversion efficiency, we plot the figure of merit (FOM) $\Gamma_{\pm} = \frac{\gamma^2 |b(\omega_0 \pm \Omega)|^2}{|s(\omega_0)|^2}$ that indicates the fraction of incident energy converted to the desired frequency in Figure 3.

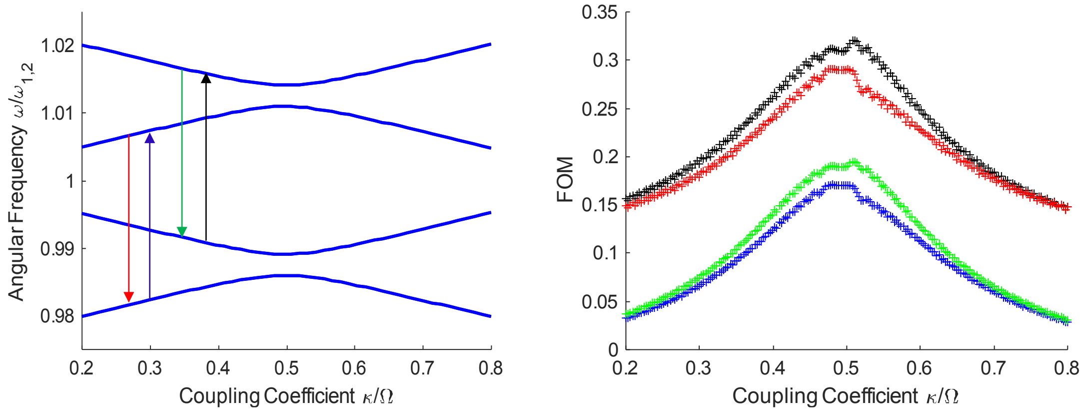

Figure 3: Band-like structure of the Floquet modes and arrows showing transitions of interest (left). Figures of merit corresponding to these transitions. The conversion efficiency is enhanced when the incident mode is a 0-order mode and at the band edge ($\kappa = \Omega/2$).

Selective Frequency Conversion

Depending on the parameters, the frequency conversion mechanism can be quite different. For example, if we plot the FOMs $\Gamma_{\pm}$ as a function ofκ (in this case at the first bandgap, $\kappa = \Omega/2$ and at the first accidental degeneracy $\kappa \cong \Omega$) we notice that the combs produced are asymmetric for $\kappa = \Omega/2$ (first bandgap, Fig. 4 left) and symmetric for $\kappa \cong \Omega$ (first crossing, Fig. 4 right).

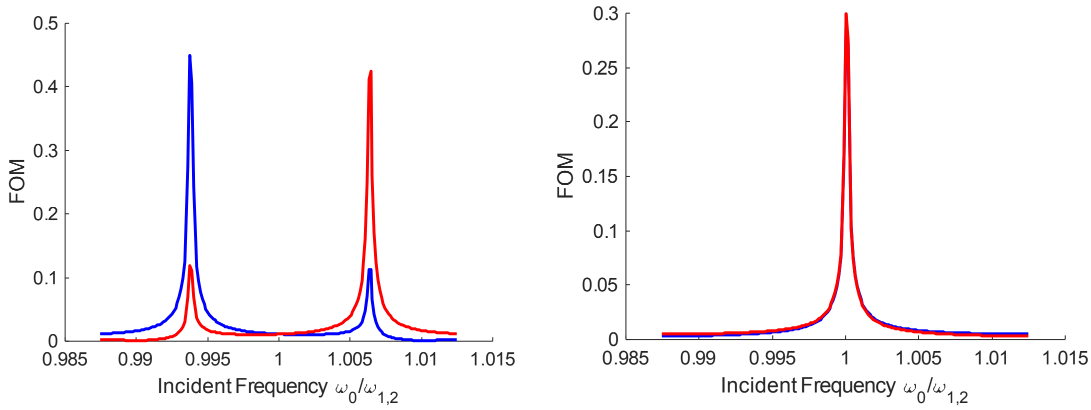

Figure 4: FOMs Γ_+(blue) and Γ_- (red) for different incident frequencies ω_0. The frequency conversion is asymmetric for $\kappa = \Omega/2$ (left) and symmetric for $\kappa \cong \Omega$ (right). The enhancement in conversion efficiency occurs when ω_0 is equal to a Floquet mode frequency.

This behavior indicates that the system is highly tunable, and we achieve various regimes of frequency conversion. Depending on the system parameters, the frequency conversion can either be symmetric ($\Gamma_+ = \Gamma_-$, Fig. 5 right) or asymmetric (for example $\Gamma_+ \ll \Gamma_-$, Fig. 5 left). A simplified model that takes into account only 3 frequency components is able to estimate with a good precision the frequency components of $a(\omega)$ and $b(\omega)$. These estimations are the red dotted lines in Figure 5, while the blue lines are the numerical solution of the dynamic coupled mode equations.

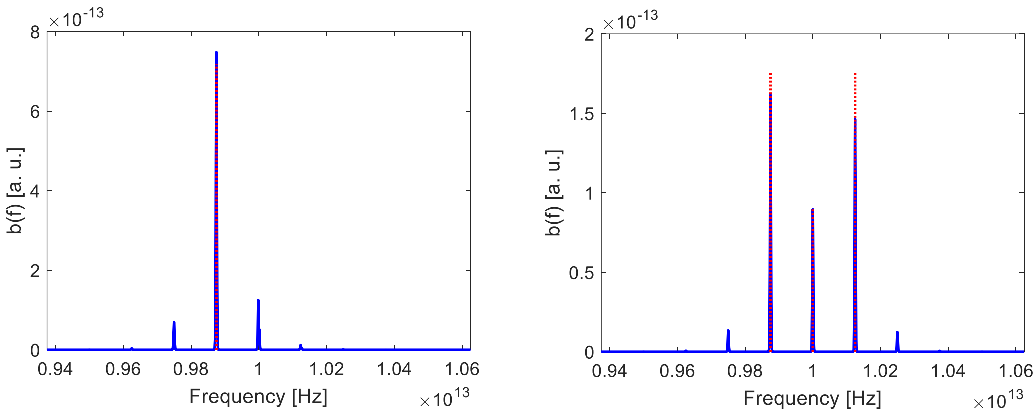

Figure 5: Frequency combs obtained at $\kappa = \Omega/2$ (left) and symmetric for $\kappa = \Omega$ (right). The simple 3 frequency model gives a good estimation of the frequency conversion efficiency.

Thus, by using a different coupling parameter κ, one can achieve various types of frequency conversion. Since the system has many tunable parameters it is possible to selectively enhance any frequency of the comb by choosing these parameters accordingly.

Conclusion

We present a way to achieve selective frequency conversion that exploits time modulated coupled cavities. We show that the Floquet modes play a crucial role in the conversion efficiency. It is also possible to select the frequency components that will be more efficiently converted by this process. This mechanism is fairly general and can be employed in a wide range of physical resonances.

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References

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