A poor man's coherent Ising machine: Solving optimization problems with opto-electronic feedback systems

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Coherent Ising machines are a promising concept to replace digital computers in computationally challenging tasks such as combinatorial optimization. We present a new concept for CIMs that generates Ising spin networks through a feedback-induced bifurcation [1]. Contrary to current state-of-the-art CIMs, we can circumvent the necessity for nonlinear optical processes or large external cavities, which offers significant advantages regarding stability, footprint and cost. This allows us to build a CIM as a compact design that can be built from just a few off-the-shelf components. We benchmark our setup with MAXCUT optimization problems and find that performance is similar or better than state-of-the-art CIMs and quantum annealers.

Combinatorial optimization problems are a part of our everyday life and are present in common tasks such as traffic routing, scheduling of flights or soccer games and power grid optimization. Optimization algorithms have therefore become responsible for ensuring that many aspects of our society can run as efficiently and as sustainable as possible. However, most combinatorial optimization tasks are known to be NP-hard problems, which makes them difficult to solve on conventional digital computers. As a consequence, algorithms solving large-scale optimization problems often have to be executed on large computer clusters and hence consume substantial amounts of computational resources in the process, which raises significant issues in regard to cost and sustainability. In order to increase the speed and lower the cost and energy consumption for solving optimization problems, various new unconventional computing schemes such as quantum annealing are thus being investigated to replace digital computers in optimization tasks.

A promising concept that has received considerable attention are coherent Ising machines (CIMs). Similar to quantum annealing, CIMs solve optimization problems by mapping their cost function to the energy function of the Ising model [2]. The optimal solution is then obtained by finding the ground state of the implemented Ising model, which is performed by an artificial Ising spin network that can be generated from various optical system such as coupled lasers or degenerate optical parametric oscillators (DOPOs) [3,4]. CIMs based on DOPOs in particular are able to find optimal solutions for large scale optimization problems within just a few milliseconds [4,5,6], which can be significantly faster than conventional optimization algorithms [4]. However, the generation and manipulation of DOPOs has proven to be rather challenging, since it requires kilometer-long fiber cavities, phase-sensitive detection, nonlinear optical materials and powerful laser systems. As a consequence, current-state-of-the-art CIMs typically consist of complex and costly setups, that have large footprints and are highly susceptible to perturbations. We try to address these issues by proposing a new approach for generating artificial Ising spin networks [1]. Contrary to DOPO-based CIMs, spins are generated through a feedback induced bifurcation in a Mach-Zehnder modulator, hence circumventing the necessity for complex optical setups required for the DOPO generation. This results in a compact and inexpensive CIM, that can be built from a few off-the-shelf components and is significantly more stable than current state-of-the-art machines.

Implementing Ising spin networks with opto-electronic feedback systems

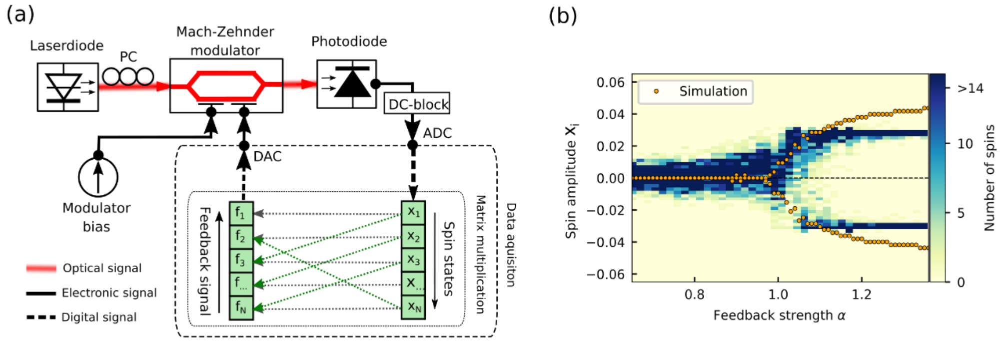

Figure 1: (a) Experimental setup of an OEO-based CIM. (b) Distribution of spin amplitudes for N=100 uncoupled OEOs as the feedback strength is increased. Orange dots indicate the maximum of the distribution obtained from simulating eq. (2). Adapted from [1].

CIMs work by mapping analog optical bistable states $x_i = \{-x_0, x_0\}$ to a network of Ising spins $\sigma_i = \{-1,1\}$. For optical systems undergoing a pitchfork bifurcation, the Ising model is implemented by associating the stable fixed points $x_i = \{-x_0, x_0\}$ with the “spin up” ($\sigma_i = 1$) and “spin down” ($\sigma_i = -1$) configurations. By coupling the states through the coupling matrix J_{ij}, the gain of the system becomes proportional to the energy function of the Ising model

$$H_{Ising} = \sum_{ij} J_{ij}\sigma_i\sigma_j \qquad (1)$$

and local energy minima of the Ising Hamiltonian become fixed points of the coupled optical system. When driving the optical system close to the origin of the pitchfork bifurcation, only the fixed points associated with the global minimum become stable, which allows for efficient computation of the optimal solution for various combinatorial optimization problems [2,4,5,6]. The challenge of building CIMs arises from finding a fast, optical system that exhibits a perfectly symmetric pitchfork bifurcation. Here, we propose to utilize opto-electronic oscillators (OEOs) to facilitate the required bifurcation. OEOs are an attractive choice due to their rich bifurcation structure and inherent stability. OEOs can also be assembled into large networks through time multiplexing schemes. The time-discrete dynamics of such an OEO network x_i are governed by the following iterative map:

$$x_i[n+1] = cos^2\left(\alpha x_i[n] + \beta \sum_j J_{ij} x_j[n] + \xi_i[n] - \frac{\pi}{4}\right) - 0.5 \qquad (2)$$

Here, α is the strength of the self-feedback while β determines the coupling strength. An additional noise term ξ_i accounts for noise in the system and a constant bias of $-\pi/4$ is applied to the nonlinearity. Additionally, the DC component of the system is removed by subtracting -0.5. It can easily be shown that this map approximates a pitchfork bifurcation close to the bifurcation point at $\alpha = 1$, so that large artificial Ising spin

networks can be realized by coupling several OEOs together [1]. Fig.1a shows our setup for implementing such an OEO-based CIM. The output of a single-mode laser is channeled through a Mach-Zehnder modulator (MZM). During one iteration, the Ising spins are generated from the time-multiplexed input signal to the MZM before being detected by a photodiode. The DC-component of the signal is removed by a DC-block before being digitized by an ADC. The feedback signal is then generated inside of an FPGA. The FPGA is responsible for de-multiplexing the detected signal, performing the vector-matrix multiplication for the coupling term and multiplexing the spin states into the feedback signal. This feedback is then transformed into an analog signal by a DAC before being injected back into the input of the MZM to close the feedback loop.
To test the ability of our setup to facilitate a pitchfork bifurcation, we implement a network of $N = 100$ uncoupled OEOs ($\beta = 0$). Fig. 1b shows the amplitude distribution as the feedback strength is increased. We find that our setup reproduces the expected bifurcation and agrees well with the expected results obtained from simulations of eq. (2). Above the bifurcation point, we observe that the probabilities for spins to be up or down are almost equal, which is to be expected from independent Ising spins and thus highlights that the bifurcation is symmetrical.

Benchmarking with MAXCUT and spin-glass optimization problems

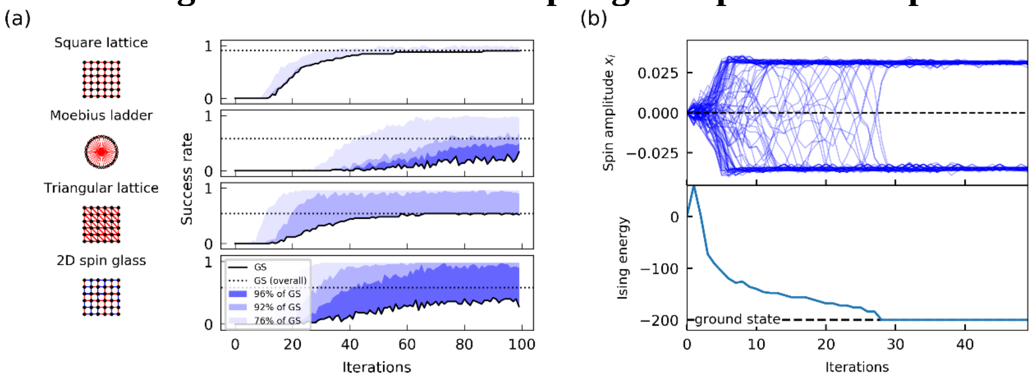

Figure 2: (a) Time evolution of the success rate to reach the ground state for four different graph structures with N=100 spins. (b) Time evolution of spin amplitude and Ising energy during an exemplary ground state search of a square lattice. The ground state is reached after 28 iterations. Adapted from [1].

To implement optimization problems, an appropriate coupling matrix J_{ij} has to be chosen. There exists a variety of different ways to map various optimization problems to a specific coupling matrix. Here, we want to focus on the MAXCUT problem, which is the mathematical task of dividing a graph into two new subgraphs by cutting through the maximal number of connecting links. For various different graph structures, MAXCUT is known to be an NP-hard problem, which makes it a common benchmark. To implement MAXCUT with the Ising model, J_{ij} is equivalent to the connectivity matrix of the graph with each non-zero edge set to be antiferromagnetic ($J_{ij} = -1$). We test the ability of our setup to find the optimal solution for various graph structures with N=100 spins which are shown in figure 2a. For each graph, we measure the time evolution of the success rate, which is the percentage of instances in which the optimal solution is found. As a first instance, we investigate a simple 10 by 10 square lattice with periodic boundary conditions, where all spins are aligned in a grid and coupled to their four nearest neighbors. After 100 iterations, an overall success rate of 90 percent is attained. The remaining calculations all fall within 76 percent of the optimal solution (indicated by shades of blue in fig. 2a), which still presents a good approximation. To better demonstrate the working principle of the CIM, we show the time evolution of the spin amplitude x_i and the respective Ising energy calculated from eq. (1) during an exemplary ground state search in fig. 2b. The CIM is always started in the unstable fixed point $x_i\,[n = 0] = 0$. As the computation progresses, the spins start to bifurcate into the upper and lower fixed points. During the bifurcating, the coupling leads to a reordering of the spins, which is accompanied by a drop of the Ising energy. After around 30 iterations, the reordering stops as a stable configuration is reached. At this point, the global minimum has been found so that the Ising energy is at $E_{Ising} = -200$.

We also consider more complex graphs, where competing spin interactions cause lattice frustrations, which is known to increase the likelihood of becoming trapped in local energy minima. For the moebius ladder graph, spins are arranged in a ring structure and also coupled to their opposing neighbors, which causes lattice frustrations if $N/2$ is an even number. We find that after 100 iterations, the CIM reaches an overall success rate of 59 percent (indicated by dashed black line), which includes cases where the machine was able to escape from the global optimum due to noise. Compared to previous results obtained by other state-of-the-art CIMs [6], this is almost a threefold increase in the success rate. The same is also true for a 2D spin glass, where the connecting links are randomly chosen to be ferromagnetic ($J_{iji} = 1$) or antiferromagnetic ($J_{ij} = -1$). Here, we attain a success rate of 58 percent, which is almost twice that of previously reported results for DOPO-based CIMs [5]. A similarly high success rate can also be observed for the triangular lattice with an overall success rate of 52 percent. It is important to note that even in the cases where the ground state was not reached, we find that the solutions obtained by the CIM still present good approximations that are close to the optimal solution (mostly within 92 percent of the optimum).

In conclusion, we have proposed and demonstrated a new concept for CIMs based on opto-electronic feedback systems. We show how this concept can be realized as a compact and inexpensive photonic setup and find that the performance in solving MAXCUT optimization problems is equal or even superior to other state-of-the-art CIMs. An important reason for this overall good performance is linked to the inherent stability of our setup. Since the OEO-based CIM foregoes the phase-sensitivity present in DOPO-based CIMs, our setup is significantly more robust and is thus easier to drive to its ideal operating point close to the bifurcation point. This general type of feedback system can be implemented with a wide variety of different systems such as analog electronic circuits, which makes our proposed system a promising alternative to existing CIM concepts and could lead the way to compact, inexpensive and performant CIMs.

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