Temperature-induced stochastic resonance in time-modulated Kerr non-linear photonic cavities

B. Braeckeveldt[1], B. Maes[1]

[1] Micro- and Nanophotonic Materials Group, Research Institute for Materials Science and Engineering, University of Mons, 20 Place du Parc, Mons B-7000, Belgium

Driven photonic cavities are widely studied because they exhibit many interesting effects such as non-reciprocity and cooling. Furthermore, adding noise in a modulated non-linear system can lead to stochastic resonance which corresponds to periodic transitions between stable states, potentially resulting in enhanced energy harvesting. In this work, we study coherent and incoherent outgoing power from a non-linear driven photonic cavity coupled to an external port. Using a Langevin framework and temporal modulation, we show that the system temperature induces stochastic resonance in the bistable regime. We extensively explore various cases depending on the system temperature, loss rates and modulation frequency. We demonstrate that such a system exhibits frequency conversion, maximized at the stochastic resonance.

Introduction

In this paper, a photonic cavity with three aspects is considered: non-linearity, time-modulation and noise. Temporal modulation in photonic structures is an interesting subject as it leads to numerous phenomena such as optical isolation[1], reciprocity breaking[2] or even cooling[3]. On the other hand, stochastic resonance is a process where the addition of noise in a modulated bistable system can increase signal amplitude by achieving synchronicity between state transitions frequency and modulation frequency[4]. This can be performed either by varying the modulation period or noise amplitude. This phenomenon is extensively studied, and has been recently employed for energy harvesting enhancement[5]. In what follows, we consider a numerical model describing a single non-linear photonic cavity. It is based on temporal-coupled mode theory. We first present the numerical set-up with all the parameters and subsequently, we present how stochastic resonance can be used and optimized for frequency conversion.

Numerical model

The non-linear photonic cavity is coupled to an external channel as depicted in Fig. 1. In this system, the cavity mode a is described using Langevin and coupled-mode theory frameworks. The equations defining the mode evolution are[6-7]

$$\begin{cases} \dfrac{da}{dt} = [j(\omega_0 - \alpha|a|^2) - \gamma]a + \sqrt{2\gamma_d}\,\xi_d + \sqrt{2\gamma_e}\,s_+, \\ s_+ = s_p e^{j\omega_p t} + \xi_e, \\ s_- = -s_+ + \sqrt{2\gamma_e}\,a, \end{cases} \tag{1}$$

where $|a|^2$ provides the mode energy and $|s_+|^2$ and $|s_-|^2$, the incoming and outgoing power from/into the external port respectively. The input power consists of both a monochromatic pump at frequency ω_p (close to the resonance frequency ω_0) and thermal radiation ξ_e arising from the external bath at temperature T_e. Coupling between cavity and port is controlled via the decay rate γ_e. On the other hand, γ_d represents the internal dissipation of the cavity at temperature T_d. Non-linearities are managed through the parameter α chosen to be real and positive. In that way, the system only exhibits self-phase modulation (SPM) but not two-photon absorption (TPA)[8]. In equation (1), ξ_e and ξ_d are two independent delta-correlated noise sources arising from external and internal bath temperatures respectively and satisfying,

$$\langle \xi_i^*(t)\xi_i(t')\rangle = k_B T_i \delta(t-t'), \qquad i\in\{e;d\}, \tag{2}$$

where $\langle . \rangle$ means a "thermodynamic ensemble average". This coupled system exhibits bistability under certain conditions. Defining the total dissipation rate $\gamma=\gamma_e+\gamma_d$, the dimensionless detuning $\Delta=\frac{\omega_p-\omega_0}{\gamma}$ and the effective non-linear coupling $\zeta=\alpha|s_p|^2\gamma_e/\gamma^3$, one can show that these requirements are $\Delta<-\sqrt{3}$ and $\zeta_1<\zeta<\zeta_2$ where ζ_1 and ζ_2 are functions of the detuning. We extend this model by applying temporal modulation through the pump with the addition of a signal at frequency $\omega_p+\Omega$ and of small amplitude compared to the initial pump[9]. In such a situation, the system remains driven at frequency ω_p but is now modulated at frequency Ω. As a result, the non-linear coupling changes with time as the pump amplitude varies. As equation (1) belongs to the category of stochastic differential equation (SDE), it can not be solved using classical numerical solvers[10]. We use a homemade solver based on the Runge-Kutta method that we have validated by recovering results presented in previous studies.

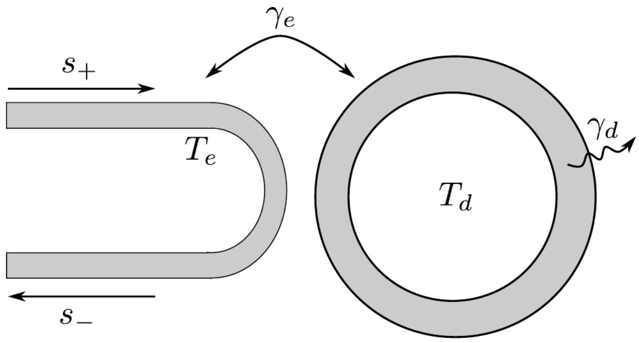

Figure 1: Structure of the simulated model.

Results

The system described by equation (1) is bistable in the simulated conditions. As the system potential energy is modulated through the small signal added to the pump, the addition of noise can induce hopping between both stable states. In equation (1), the noise intensity is controlled by both temperatures, the external one and the cavity one. We show typical time traces (Fig.2a-d), corresponding to four situations indicated in Fig.2e. If the noise intensity is too small Fig.2a, corresponding to low temperatures, the transition probability is close to zero and the system will remain in one of the two modulated states. As the temperature increases, the transition probability rises and the system can jump from one state to the other (Fig.2b). Synchronization between the modulation and transition period occurs for specific temperatures corresponding to stochastic resonance (Fig.2c). In this situation, the relative output power (output over input) is maximum as depicted in Fig.2e. For higher temperatures, it becomes very easy for the system to overcome the potential barrier and transitions occur at a highest frequency than the modulation one, destroying synchronicity Fig.2 d). We found that both temperatures play the same role when considering the outgoing power. Indeed, putting the external temperature to zero and varying the internal one or vice-versa, provides the same variation of the outgoing power. Furthermore, if one wants to keep both temperatures equal ($T=T_e=T_d$), stochastic resonance appears for a temperature twice as low as in the situation where one of the temperatures equals 0K.

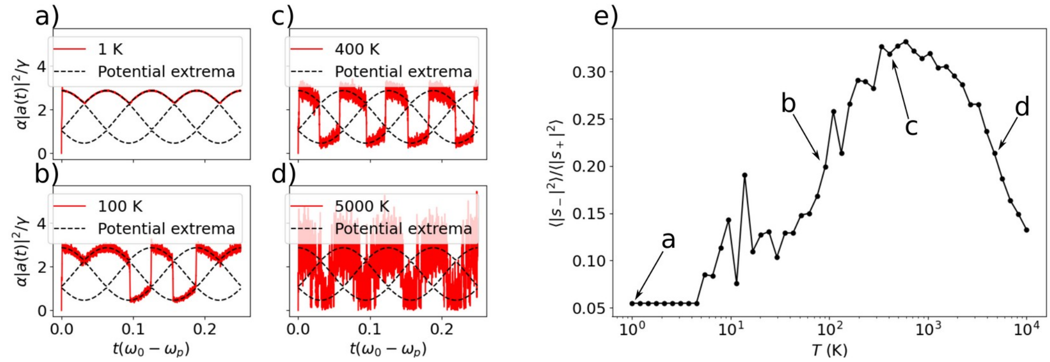

Figure 2: Time traces of dimensionless mode energy a-d) and associated relative output power e).

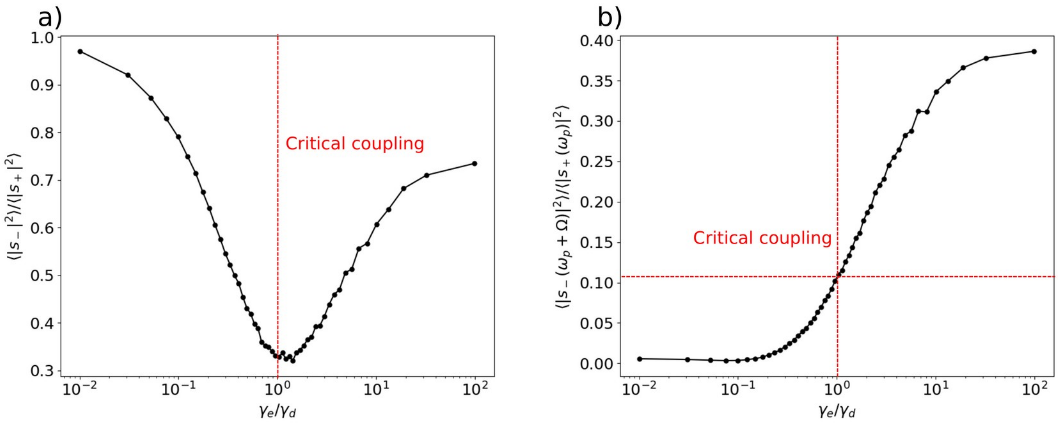

Figure 3: Up-conversion efficiency a) for various ratio between external and internal loss. Corresponding relative output power b).

As the system is modulated, the power spectral density of the ingoing power is mainly composed of a peak at the pump frequency and a second, much weaker, at the signal frequency, therefore shifted by the modulation frequency with respect to the pump. We decided to look at the outgoing power for these two frequencies to study how stochastic resonance affects the system and how much power is converted from the pump frequency to the signal one. At critical coupling, when both dissipation rates are equal, we observe that almost all the incoming power is absorbed by the cavity and the outgoing power is minimal (Fig.3a). In this situation the output at signal frequency becomes orders of magnitude larger than the input power at the same frequency. Moreover, looking at frequency conversion, i.e. the amount of outgoing power at frequency $\omega_p+\Omega$ compared to the input power at frequency ω_p (Fig.3b), we obtain a conversion of around 10%. To increase the up-conversion efficiency, one can consider a coupling factor much larger than the internal dissipation rate ($\gamma_e \gg \gamma_d$) as show in

Fig.3b. By taking a coupling factor two orders of magnitude larger than the internal dissipation rate, the conversion efficiency reaches 40%. On the contrary, if the external dissipation rate is small compared to the internal one, the conversion is very weak (close to zero) because the external channel does not couple with the cavity and outgoing power equals the incoming one.

Conclusion

Using coupled-mode theory and Langevin frameworks, we show that by changing the system temperatures, it is possible to tune the outgoing power. For a given temperature, the system reaches stochastic resonance, maximizing its output power. External and internal baths play the same role and it is therefore possible to keep the same temperature for the external channel and the cavity. At critical coupling, the output power is minimum because almost all the incoming power is absorbed by the cavity. This situation should not be used for a good up-conversion efficiency. We show that it can be optimized by considering a large coupling factor compared to the internal dissipation rate.

Acknowledgments

We thank Pr. Alejandro Rodriguez for precious discussions and advice. This work was supported by the Fonds pour la Formation à la Recherche dans l'Industrie et dans l'Agriculture (FRIA) and by the Fonds National de Recherche Scientifique (FRNS) in Belgium. Computational resources have been provided by the Consortium des Équipements de Calcul Intensif (CÉCI), funded by the Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under Grant No. 2.5020.11 and by the Walloon Region.

References

[1] Z. Yu and S. Fan, 'Complete optical isolation created by indirect interband photonic transitions', Nature Photonics, vol. 3, no. 2, Art. no. 2, 2009.

[2] D. L. Sounas and A. Alù, 'Non-reciprocal photonics based on time modulation', Nature Photonics, vol. 11, no. 12, pp. 774–783, 2017.

[3] S. Buddhiraju, W. Li, and S. Fan, 'Photonic Refrigeration from Time-Modulated Thermal Emission', Phys. Rev. Lett., vol. 124, no. 7, p. 077402, 2020.

[4] L. Gammaitoni, F. Marchesoni, E. Menichella-Saetta, and S. Santucci, 'Stochastic Resonance in Bistable Systems', Phys. Rev. Lett., vol. 62, no. 4, pp. 349–352, 1989.

[5] K. J. H. Peters, Z. Geng, K. Malmir, J. M. Smith, and S. R. K. Rodriguez, 'Extremely Broadband Stochastic Resonance of Light and Enhanced Energy Harvesting Enabled by Memory Effects in the Nonlinear Response', Phys. Rev. Lett., vol. 126, no. 21, p. 213901, 2021.

[6] C. Khandekar, Z. Lin, and A. W. Rodriguez, 'Thermal radiation from optically driven Kerr ($\chi^{(3)}$) photonic cavities', Appl. Phys. Lett., vol. 106, no. 15, Art. no. 15, 2015.

[7] C. Khandekar, A. Pick, S. G. Johnson, and A. W. Rodriguez, 'Radiative heat transfer in nonlinear Kerr media', Phys. Rev. B, vol. 91, no. 11, p. 115406, 2015.

[8] A. Rodriguez, M. Soljačić, J. D. Joannopoulos, and S. G. Johnson, '$\chi^{(2)}$ and $\chi^{(3)}$ harmonic generation at a critical power in inhomogeneous doubly resonant cavities', Opt. Express, vol. 15, no. 12, pp. 7303–7318, 2007.

[9] M. I. Dykman, D. G. Luchinsky, R. Mannella, P. V. E. McClintock, N. D. Stein, and N. G. Stocks, 'Supernarrow spectral peaks and high-frequency stochastic resonance in systems with coexisting periodic attractors', Phys. Rev. E, vol. 49, no. 2, pp. 1198–1215, 1994.

[10] P. E. Kloeden and E. Platen, "Numerical Solution of Stochastic Differential Equations", Berlin, Heidelberg: Springer, 1992.