Bragg gratings for phase-mismatch compensation in phase-sensitive Four-Wave Mixing

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*Abstract: Bragg gratings (**BG**) are resonant structures that exhibit a strong dispersion close to their transmission stopband, enabling wave-vector modifications. Integrated BG can be made by coupling evanescently from a straight waveguide to an auxiliary periodic structure. This can be exploited either to enable phase-mismatch compensation in an OPA [1], or quasi-phase matching in hybrid waveguides [2]. Here, we develop a method to design a grating that allows phase-mismatch compensation for a Bragg-Scattering Four-Wave-Mixing process (**BS-4WM**). As a result, we can predict by means of numerical simulations that this grating-assisted phase-matching allows unprecedented conversion bandwidth. We also design structures allowing an experimental demonstration with existing technologies.*

Four-Wave Mixing (4WM) finds applications in frequency conversion [3] or quantum entanglement [4] and Bragg-Scattering Four-Wave Mixing (BS-4WM) is a specific 4WM process that is phase-sensitive. BS-4WM is therefore theoretically noise free as it transfers directly energy from signal to idler waves as depicted in figure 1. It can preserve quantum states during the conversion [5] and has been used for optical switches [6]. BS-4WM has already been demonstrated in highly nonlinear fibers [7] and SiN waveguides [8]. Fibers exhibit a low nonlinear coefficient γ, so that meters of fiber are typically needed to observe an efficient conversion. Meanwhile, waveguides have a higher γ and typically sub-meter interaction lengths, hence lower accumulated dispersion. In both cases, efficient conversion requires to satisfy the phase-matching condition: $\Delta\beta L < 2\pi$. This condition is expressed as a function of the wave vectors (momenta) $\beta_k = n(\omega_k)\frac{\omega_k}{c}$,

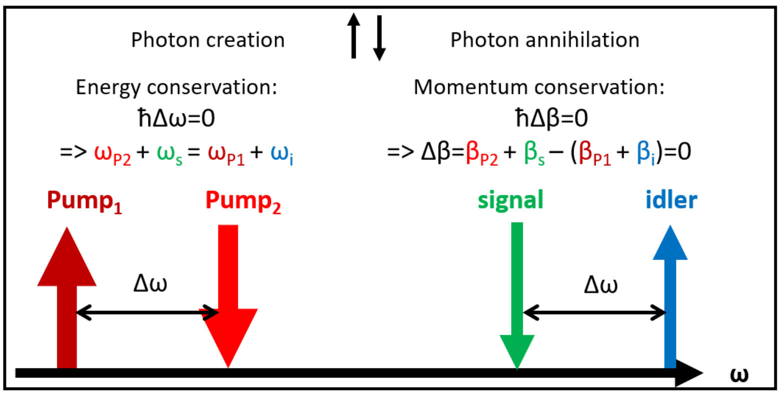

Figure 1. Bragg scattering Four Wave Mixing. Two intense light fields ("pump" fields, at ω_{P1}, ω_{P2}) and a weaker one (at ω_s) propagate inside the material. Two photons (at ω_{P2}, ω_s) are annihilated and two photons

are generated (at ω_{P1}, ω_i) while satisfying energy and momentum conservation laws: $\hbar\Delta\omega = 0$ and $\hbar\Delta\beta=0$ with $\Delta\omega = (\omega_{P1} + \omega_i) - (\omega_{P2} + \omega_s)$ and the phase mismatch $\Delta\beta = (\beta_{P1} + \beta_i) - (\beta_{P2} + \beta_s)$.

with n the refractive index associated to each wave frequency $\omega_{P1}, \omega_{P2}, \omega_s, \omega_i$, and is expressed as $\Delta\beta = (\beta_{P1} + \beta_i) - (\beta_{P2} + \beta_s)$ in the configuration depicted in figure 1. In that typical configuration, the pump wavelengths and the single photon wavelengths have to be on opposite sides of the 0-group velocity dispersion (0-GVD) wavelength. That condition can be challenging or even impossible for some set of wavelengths due to the limited control on the 0-GVD by dispersion engineering. Indeed, if phase engineering has been extensively applied for optimization of 4WM in SiN and Si waveguides, it typically requires affecting both the width and the thickness of the core materials. This is not always possible because of fabrication constraints, or bare availability of the process for multi projects wafer run (MPW) services. The influence of a Bragg Grating on the generation of solitons has been studied more than two decades ago [9], while its positive effects on OPA operation was reported more recently [1]. Non-resonant gratings have also been exploited, including in four wave mixing demonstrations [2]. Similarly to dispersion engineering used for 4WM [10,11], we engineer a grating to allow phase matching of a given BS-4WM process.

The wavelength conversion efficiency η via BS-FWM can be simply expressed as [5]

$$\eta = |2\gamma PL|^2 \text{sinc}^2(\sqrt{(2\gamma P)^2 + \Delta\beta^2}L) \quad \text{(eq. 1)}$$

when pump powers P are equal [12], γ is the non-linear coefficient, L the interaction length and $\Delta\beta$ the phase-mismatch. The conversion is unity when $\gamma PL = \frac{\pi}{4}$ and $\Delta\beta = 0$. This maximum conversion can be hard to reach. Indeed, choosing the frequencies of photons that interact will constrain the phase-mismatch $\Delta\beta$, as the wave vectors are function of the frequency: $\beta = n(\omega)\frac{\omega}{c}$, with n the optical index. A Bragg grating exhibits a spectral domain (bandgap) where transmission is forbidden around a resonance frequency $\omega_{res} = \frac{\pi c}{n_{mean}\Lambda}$ with the grating period Λ and the mean index n_{mean}[13]. Outside of this bandgap, the grating has a strong influence $q(\omega)$ on the wave vector β of the propagating fields. It is defined in a linear regime by:

$$q^2(\omega) = \delta^2 - \kappa^2 \quad \text{(eq. 2)}$$

With $\delta = \beta(\omega) - \frac{\pi}{\Lambda}$ and the strength of the grating $\kappa \approx \frac{2\pi}{\lambda}dn$, dependent of the index modulation amplitude dn. In practice, $q(\omega)$ is calculated using a Taylor expansion around the resonance frequency ω_{res} [13]. These analytical formulas can be included in the expression of effective wave vectors

$$\beta eff = \beta + q(\omega) \quad \text{(eq. 3)}$$

as long as the nonlinear interaction is weak enough. Indeed, q should be corrected for the Kerr-induced nonlinear phase: $q = -\frac{\kappa(1-f^2)}{2f} - \frac{\gamma P}{2}\frac{(1-f^2)}{1+f^2}$ and $\delta = -\kappa\frac{(1+f^2)}{2f} - \frac{3\gamma P}{2}$, with f a real parameter that quantifies the energy distribution between transmitted and reflected fields. We have $|q_{non\,linear} - q_{linear}| = \frac{\gamma P}{2}\left|\frac{1-f^2}{1+f^2}\right|$. Because $h(x) = |\frac{1-x^2}{1+x^2}|$ maps the real numbers onto [0;1], $h(x) \leq 1$ so we deduce that $|q_{non\,linear} - q_{linear}| \leq \frac{\gamma P}{2}$ irrespective of the details of the grating or the wavelength detuning between the field and the grating resonance. As $P = \frac{\pi}{4\gamma L}$ for unity efficient BS-4WM, we have that $\Delta q = |q_{non\,linear} - q_{linear}| \leq \frac{\pi}{8L}$. That nonlinear correction is significantly smaller than the phase mismatch that we can accept: $\Delta q \ll 2\pi/L$. Thus, the linear expression for the grating-induced dispersion is a good approximation in our case. Therefore, equations (2) and (3) can be combined to seek the proper phase matching $\Delta\beta < 2\pi/L$ demanded

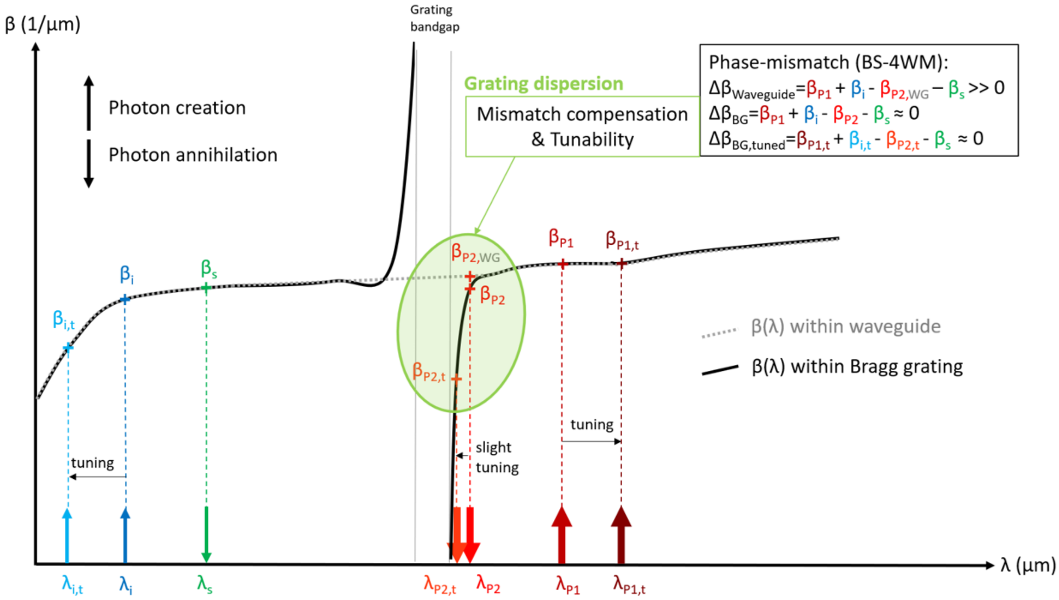

Figure 2. Schematic example of BS-4WM in presence of a grating. The light grey dotted line represents the waveguide dispersion $\beta(\lambda)$ without any grating, while the continuous black curve represents the dispersion in the presence of the grating. Close to the bandgap (vertical thin gray lines) the grating resonance change $\beta(\lambda_{P2})$ and allows phase-matching. If we want to tune some wavelengths of the system (ex. λ_i & λ_{P1}), the newly induced mismatch can be compensated by slightly tuning λ_{P2} resulting in a high tunability of the system.

by eq 1. Hence, for a typical 1cm-long grating, and $\gamma PL = \frac{\pi}{4}$, the linear formula of β_{eff} gives an error of 0.4 cm^{-1} much smaller than the phase mismatch bound of 6.28 cm^{-1}.
We now can examine a practical situation for which the grating resonance is close to the pump field at ω_{P1} . In that scenario, β_{P1} undergoes the grating influence so that $\beta_{P1,eff} = \beta_{P1} + q(\omega)$ while other waves remain unaffected by the presence of the grating (principle illustrated in figure 2). One can then express the modified phase mismatch as $\Delta\beta_{BG} = (\beta_{P1,eff} + \beta_i) - (\beta_{P2} + \beta_s) = \Delta\beta + q(\omega)$ implying that the influence of the grating $q(\omega)$ directly comes in the phase mismatch.
To illustrate the benefit of this design, we focus on a particular configuration involving pump fields at λ_{P1}=2090 nm and λ_{P2}=1750 nm with an initial signal photon at λ_s=1550 nm and a final idler wavelength λ_i=1350 nm. If we allow unrestricted dispersion engineering via the width and thickness of the waveguide, the phase mismatch for that process is minimized for a cross section of 1.25μm x 0.9μm (width x height) which correspond to a phase mismatch of $\Delta\beta$=0.15 cm^{-1} < $2\pi/L$ =6.28 cm^{-1} for a 1cm-long interaction. However, the tunability of the process is low: a change of ω_s requires a similarly large change of ω_{P1}. As an example, if we allow tuning of the pump over ±0.5nm (something typical from a semiconductor laser diode), the signal can be tuned by 1.2 nm.
For a grating assisted waveguide, we are starting from a standard monomode waveguide cross section (as available via common MPW services) of 1μm x 0.8 μm. Without accounting for the grating, the phase mismatch would be 90 cm^{-1}. This can be brought down to 0.26 cm^{-1} thanks to a grating with an index modulation $dn = 2{,}4.10^{-3}$ and a resonant wavelength $\lambda_{res} = 2091.296$ nm. Not only this offers more fabrication freedom but also improves the tunability. Because $\beta_{P1,eff}$ exhibits a resonant behavior, a slight

tuning of λ_1 will indeed results in a larger impact on phase-mismatch than in the previous case (similarly to figure 2). More specifically, a tuning of the pump wavelength by 0.1 nm provides signal tunability over 30 nm.
Without the presence of the grating, such a tuning would require the pump to be modified by a similar amount thus requiring a change of technology for the laser (external cavity laser or OPO). A demonstration is under investigation using a Bragg grating made from equally spaced pillars evanescently coupled to a straight SiN waveguide (1µm x 0.8µm). We then expect $\Delta\beta \leq$ 0.60 cm^{-1} for wavelengths $\lambda_s \in$ [1530,1560] nm, $\lambda_i \in$ [1340;1360] nm, $\lambda_{P1} \in$ [2090.01; 2089.99] nm and λ_{P2}=1750 nm.

To conclude, we demonstrate that the linear formula of Bragg gratings can be used in the nonlinear regime required for efficient conversion via BS-4WM. This allows easy predictions and designs of gratings dedicated to correct a given phase-mismatch for a given BS-4WM configuration. We simulated such a grating and observed in these simulations the correction of the phase-mismatch as well as a high tunability potential.

In the future, the improvement of the acceptance bandwidth is another parameter to be maximized. The simultaneous use of several grating periods (a.k.a aperiodic gratings) might also bring additional tunability and possible other benefits. Moreover, we foresee that Bragg gratings can be exploited to forcefully mismatch spurious processes such as parametric fluorescence.

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