Improving photonic Ising machines: a guide to reaching the ground state

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With ever increasing computational demands, the need for efficient computer architectures is more and more pressing. Photonic Ising machines are a promising way to tackle challenging optimization problems. The photonic Ising machines considered here encode the variables of the problem in some property of a laser in a feedback loop, such as the phase or the intensity and evolve them dynamically. After some time, this system relaxes to a steady state, corresponding to a solution of the problem. However, obtaining the optimal solution is not always guaranteed as the system can get trapped in local minima of the cost function. Previous research has revealed that the gain of the feedback loop and the coupling strength have a great influence on being able to reach the ground state. Here, using a stability analysis, we demonstrate that by choosing the gain correctly, we can make sure that the ground state can be reached by scanning the coupling strength slowly. Interestingly, this is not possible for all physical implementations of the photonic Ising machine and we find that photonic Ising machines using a Mach-Zehnder modulator have a distinct advantage.

Introduction

Optimization problems are present in all sorts of real-life applications, from industrial design to route optimization. In recent years, the computational power harnessed to solve these problems has increased so much that its energy consumption and ecological impact can no longer be ignored [1, 2]. Therefore, the search for fast and energy efficient hardware has gained more and more attention. In this light, photonic Ising machines have arisen as a novel and promising approach to solve complex NP-hard binary optimization problems. They leverage the natural tendency of systems to settle in their lowest energy state. Many challenging binary optimization problems can be mapped to the Ising model's energy, so the ground state of this Ising problem corresponds to the optimal solution [3]. Solving the optimization problem therefore comes down to finding the ground state of the corresponding Ising system. Ising machines are physical systems dedicated to obtaining this ground state for an arbitrary Ising problem.

So far, Ising machines have been realized using various systems, including a system of superconducting qubits [4], trapped ions [5] and arrays of memristors [6]. Photonic implementations, such as systems based on spatial light modulation [7], Degenerate Optical Parametric Oscillators (DOPOs) [8] or Opto-Electronic Oscillators (OEOs) [9] are particularly interesting because of their high bandwidth and low energy consumption. In this article, we focus on these photonic implementations. More specifically, we focus on Ising machines that dynamically evolve analog variables, such as the last two implementations mentioned. For these Ising machines, the variables of the problem are encoded in the output of a laser beam, such as the phase or the intensity, in a feedback loop. The problem itself is expressed through the interaction between these variables. These photonic Ising machines let the analog variables evolve in time until they reach a steady state. This steady state corresponds to a possible solution of the binary optimization problem. However, Ising machines may not always converge to the ground state and can get trapped in local minima. The success rate of finding the ground state heavily depends on the combination of the parameters of the system and the problem at hand [10]. Surprisingly, although different Ising machine implementations are based on the same principle and share many of the same features, the success rate of the Ising machine is also dependent on the physical implementation [10]. Understanding why some problems are harder than others and how the physical implementation affects performance is therefore crucial for enhancing Ising machine efficiency.

In this paper, we employ continuation methods to investigate the ground state of benchmark problems. First, we determine how the analog spin configuration of the ground state changes when the system parameters are changed and identify three distinct difficulty classes, which align with previous observations [11]. Based on this insight, we also propose an annealing scheme to efficiently solve two of these classes. Second, we demonstrate how changing the physical implementation and appropriately selecting the gain can alter a problem's difficulty class. This could render a previously difficult problem manageable using the mentioned annealing scheme.

Model of the Ising machine

The working principle of photonic Ising machines is based on the Ising system, governed by the Hamiltonian:

$$\mathcal{H} = -\frac{1}{2}\sum_{ij}^{N} J_{ij}\sigma_i\sigma_j,$$

where N represents the total number of spins, J_{ij} denotes the coupling matrix and $\sigma_i \in \{-1, 1\}$ represent the binary spins. The photonic Ising machines considered here aim to find the ground state configuration that minimizes this Hamiltonian by dynamically evolving analog spins ($x_i \in \mathbb{R}$). After some time, the analog spins settle into a stable configuration and the corresponding binary spins are obtained by $\sigma_i = sign(x_i)$.

The specific form of the differential equations governing the photonic Ising machine's evolution depends on its physical implementation. However, they share many common features and can be described by the following general equations for the spin dynamics:

$$\frac{dx_i}{dt} = -x_i + \mathcal{F}\left(\alpha, x_i, \beta\sum_j J_{ij}x_j\right),$$

where α represents the gain, β is the coupling strength, and $\mathcal{F}$ is a nonlinear feedback function depending on analog spins, coupling strength, and gain. This nonlinear function introduces a pitchfork bifurcation when increasing the gain parameter α.

In this paper, we focus on two transfer functions: one with a third-order polynomial nonlinearity, representing DOPO-based spin dynamics [8]

$$\frac{dx_i}{dt} = (\alpha - 1)x_i - x_i^3 + \beta\sum_j J_{ij}x_j,$$

and one with a sigmoid nonlinearity to account for spin amplitude saturation in experimental setups

$$\frac{dx_i}{dt} = -x_i + tanh\left(\alpha x_i + \beta\sum_j J_{ij}x_j\right).$$

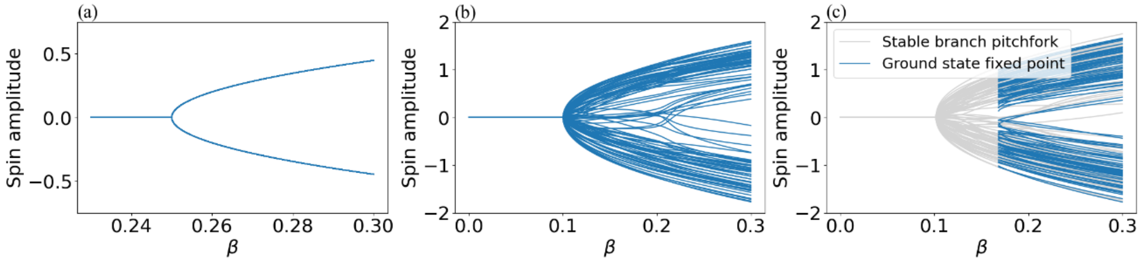

Fig. 1. Spin amplitude of all spins as a function of the coupling strength when the third order polynomial nonlinearity is used to search for the ground state of the problem with a hundred spins that interact antiferromagnetically with their nearest neighbors (a), problem g05_100.2 of the BiqMac library (b) and problem g05_100.1 of the same library (c). In (c) the gray lines show the spins of the state that would be obtained when the problem would be solved using annealing.

Simulation results

Using these differential equations, we conduct simulations of the Ising machine on benchmark MaxCut tasks, including problems with nearest-neighbor interactions and two BiqMac library problems (g05_100.1 and g05_100.2). Simulations involve initializing the analog spins at small random values and evolving them using the third order polynomial nonlinearity with fixed parameters until the system reaches the ground state. An analog spin configuration corresponding to the ground state is then used as input for a continuation algorithm, which tracks the ground state as the coupling strength β is slowly decreased.
We distinguish three difficulty classes for problems based on the behavior of the ground state when β is altered. Fig. 1 shows the amplitude of all spins as a function of β for three problems, each belonging to a different difficulty class. The problem with a hundred spins that interact antiferromagnetically with their nearest neighbors, shown in Fig. 1a, is an example of a "Spectral easy" problem, using the same terminology as in [11]. Note that for this specific problem, all spins have the same amplitude, so the lines of the spins overlap. The spins at $\beta = 0.30$ represent the analog ground state. As β decreases, the amplitude of all spins gradually becomes smaller until the state disappears through a pitchfork bifurcation at the origin. These problems are straightforward to solve since no spin flips are required to obtain the ground state configuration after the pitchfork bifurcation.
Another difficulty class are the "Ising easy" problems, exemplified by problem g05_100.2 in Fig. 1b. Once again, the spins at $\beta = 0.30$ correspond to the ground state. This time however, as β decreases, some spins do flip before the state disappears through the pitchfork bifurcation. This means that these problems are slightly more difficult to solve, but they still do not pose any challenge when using the correct annealing techniques.
A last difficulty class are the "Ising hard" problems, of which problem g05_100.1 is an example, shown in Fig. 1c. Again, the spins at $\beta = 0.30$ represent the ground state. This time however, when β is decreased, the state disappears through a saddle node bifurcation away from the origin at $\beta = 0.16$. This means the state you would obtain when using the annealing scheme used to solve Ising easy problems, shown in gray in Fig 1c, is always going to be the wrong state. Instead one has to rely on noise to accidently guide the system in the right direction. Therefore Ising hard problems represent the most challenging class to solve.
A possible solution to solve these Ising hard problems, lies in the following observation. Fig. 2 shows the spin amplitude of all analog spins of the ground state of problem

g05_100.1 as a function of β when solved using the sigmoid nonlinearity. Remember that this problem was Ising hard when using the third order polynomial nonlinearity. When using a gain of $\alpha = 0.95$, as shown in Fig. 2, the ground state has the same behavior as in Fig. 1b. The spins at $\beta = 0.030$ represent the ground state and when β is decreased, the state disappears through a pitchfork bifurcation at the origin. The problem has therefore become Ising easy for this value of α. This means the problem can be solved using the mentioned annealing schemes. This gives a distinct edge to physical implementations of Ising machines that use more complex nonlinearities, such as the photonic Ising machine based on a Mach-Zehnder modulator.

Conclusion

In summary, this study identified three distinct difficulty classes for problems tackled by Ising machines. Annealing schemes can only solve two of these classes. Furthermore, we demonstrated that a physical implementation of an Ising machine using a complex nonlinearity can shift a problem's difficulty class, potentially rendering it more easy to solve.

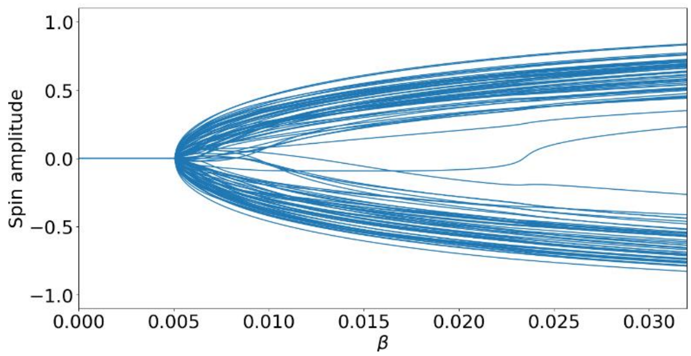

Fig. 2. Spin amplitude of all spins as a function of coupling strength of the ground state of problem g05_100.1 of the BiqMac library when using the sigmoid nonlinearity with $\alpha = 0.95$.

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