Dispersion adapted Nonlinear Schrödinger equation to simulate Kerr effects inside a Bragg grating.

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Simulation of widespread Four-Wave Mixing (4WM) inside Bragg gratings (BGs) is challenging because the rich dynamic of 4WM is incompatible with the usual coupled mode equations used to model nonlinear pulse propagation in BGs. Moreover, resolving the cm-long nonlinear system with sub-µm precision is numerically impractical. Here, we propose a simple way to tackle this issue, by solving a single NLSE with the grating information encrypted directly into the dispersion operator. This algorithm reproduces previous established results and allows for the prediction of the high tunability of Bragg scattering 4WM inside a cm-long waveguide BG.

Introduction

In quantum optics, the ability to convert the photon frequency from one operating bandwidth to another is key to interface different components. Particularly, it is required the conversion phenomenon to be phase preserving, so that the quantum information is preserved during the conversion process. The Bragg-Scattering Four Wave Mixing (BS-4WM) is a conversion process with this quality **[1]**. It involves 4 waves at 4 distinct frequencies: two powerful waves and one weak wave are injected inside a third order non-linear media (e.g. SiN) where the weak wave is turned into another one at a different frequency (**Fig.1**). For this to happen, the energy conservation and the momentum conservation have to be respected. This leads to the frequency, and wave vector of the waves to obey the following relation for the phenomenon to exist and be efficient: $\omega_1 + \omega_i = \omega_2 + \omega_s$ and $\Delta\beta = \beta_1 + \beta_i - (\beta_2 + \beta_s) < \frac{2\pi}{L}$. The subscripts 1,2 refer to the strong fields (called pumps) while *s* and *i* refer to the weak fields, the incident one usually being called "signal" and the generated one "idler". The ω refer to the angular frequency of the photon, and $\beta_k = \frac{n.\omega_k}{c}$ is the wave vector ($\beta(\omega)$ is called the dispersion), given by the angular frequency ω, the effective index *n* (at the angular frequency ω), and the speed of light in the vacuum *c*. The second relation $\Delta\beta < \frac{2\pi}{L}$ is referred as the "phase-matching condition". For the configuration of the **Fig.1**, phase matching is typically achieved when the second derivative of the dispersion (also called group velocity dispersion, GVD) at the mean frequency is zero. While some tailoring of this 0-GVD frequency is possible via the geometry of the waveguide, the material dispersion of its core and cladding still impose some limits to this dispersion engineering. Here we explore one more dispersion engineering tool, via the presence of an evanescently coupled Bragg grating. They exhibit the Bragg resonance where the wave vector is substantially modified as illustrated in **Fig.1**. This feature of Bragg gratings has been experimentally used recently in **[2]** to create an optical parametric amplifier and to produce pure quartic solitons **[3]**, which are

both third order nonlinear processes sensitive to dispersion. The simulation of the behavior of the BG in a Four-Wave Mixing (4WM) regime is challenging. There are usually two ways to simulate BG behavior.

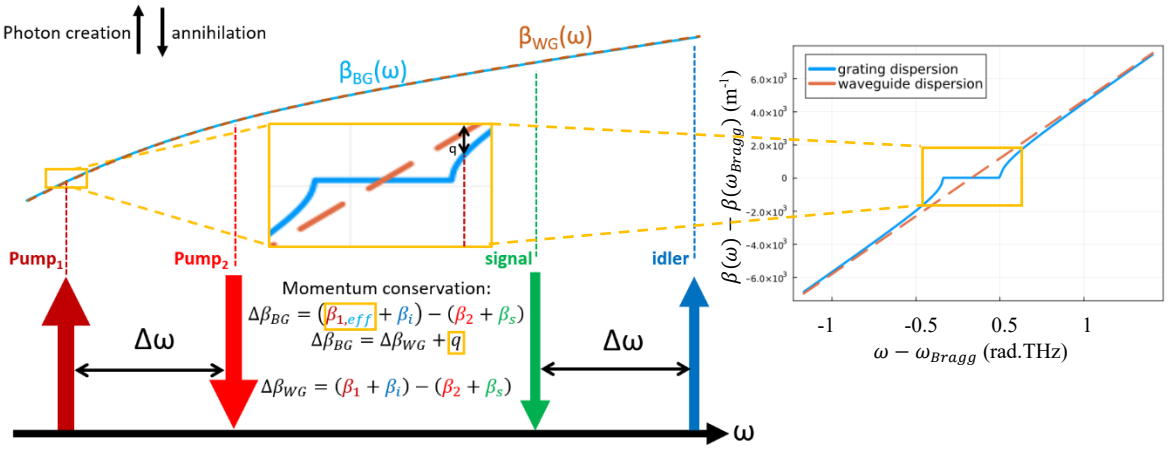

Fig.1: Left: BS-4WM inside a BG. The brown dash curve illustrates the waveguide dispersion, while the plain blue curve represents the grating one. The arrows indicate the creation/annihilation of photon (when pointing up/down) and the field intensity (a thin arrow for a weak field and inversely). The phase mismatches $\Delta\beta_{XX}$ are made from the corresponding dispersion evaluated at the photon frequencies, also called wave vector. The local influence of the BG allows for the modification of the wavevector β_1 into $\beta_{1,eff}$ which implies the modification of the phase matching condition from $\Delta\beta_{WG}$ into $\Delta\beta_{BG}$. Right: Comparison of the real part of the dispersion between a waveguide (dash line) and an evanescent grating with identical waveguide parameters. The flat part of the blue curve corresponds to the stopband of the Bragg gating, where an incident pulse is reflected. There is no propagation for the light inside this gap.

Either using the nonlinear coupled mode equation, that are valid over a narrow interval around the Bragg frequency and only take into account self-phase modulation and cross-phase modulation **[4,5]**; or use the Nonlinear Schrödinger Equation (NLSE) that is valid over a broader interval and allows for the simulation of 4WM processes, but would need uniform sections, hence a fine discretization of the system or a specific kind of grating (called uniform BG). For lengths about few cm, and grating details about the nm scale, there is 7 orders of magnitude between the NLSE interval and the size of the sample to simulate, which may be unreasonable numerically. Here we present a way to simulate a BG using a modified version of the NLSE, where the usual dispersion operator is replaced by a grating dispersion operator, which relieves the necessity to simulate over uniform sections, and allow to decrease the number of intervals to a few hundreds to perform computation. The model is tested against a previous simulation present in the literature that have been experimentally verified **[5]**. Backed by very good agreements, we use our algorithm to predict the BG to be useful to extend the tunability of BG-4WM thanks to our algorithm.

Theoretical essentials

The behavior of the Bragg grating and its influence on the overall dispersion can be modeled analytically under linear regime. These analytical formulas from the linear case are also a valid approximation when working under the optimal regime of the BS-4WM. Indeed, it can be shown the linear formulas can be used as long as the nonlinear phase $\gamma PL < 4\pi$ while the optimal regime for BS-4WM corresponds to $\gamma PL = \frac{\pi}{4}$. The values γ, P, L are standard parameters of the system, L being the interaction length, P the

geometric mean of the pump power, and γ the nonlinear coefficient. Having these formulas allows to have an analytical way to describe the dispersion of the system.
It is known that Kerr effect, such as BS-4WM, can be modeled with the NLSE, over a large spectrum span **[6]**, under the assumption that different frequencies have a group velocity mismatch that is negligible regarding the system length, and the fact the transverse spatial modes considered are similar. The first assumption is equivalent to say that the walk-off between different frequency is negligible. The second assumption means the mode profile does not differ that much from a frequency to another. In our case, we use an evanescent grating, which means modulation that create the grating takes place on the evanescent part of the field, and consider only the fundamental mode for each wavelength. It is also supposed the slowly varying envelope approximation is valid. Under the Baker-Campbell-Hausdorf approximation, the NLSE can be written as follows: $\frac{dA}{dz} = (D(t) + N)A$, with A the envelope of the field, D the dispersion operator and N the nonlinear operator. We replace the uniform dispersion operator D by the Bragg grating dispersion operator B. While B(t), D(t) can be unknown or cumbersome, B(ω) and D(ω), their Fourier transform counterpart, are very easy to express. Using β_{BG} to describe the effective wave vector that takes place in the Bragg grating, and the Taylor decomposition of the uniform dispersion $\beta = \sum_k \frac{\beta^{(k)}(\omega-\omega_{ref})^k}{k!}$, with $\beta^{(k)} = \frac{d^k\beta}{d\omega^k}(\omega_{ref})$, it is indeed possible to write $D(\omega) = \beta - \beta^{(0)} - \beta^{(1)}$and $B(\omega) = \beta_{BG} - \beta^{(0)} - \beta^{(1)}$.
Note that in the expression of B, $\beta^{(0)}, \beta^{(1)}$still refer to the uniform dispersion.
$\beta_{BG} = \beta \pm q$, q being defined as $\sqrt{\delta^2 - \kappa^2}$ with $\delta = \beta - \beta(\omega_{Bragg})$ and $\kappa \approx \frac{2\pi}{\lambda}\frac{\delta n}{2}$, δn being the index modulation. Away from the resonance, $\beta_{BG} \approx \beta$, so the change of operator will have consequences around the Bragg resonance only.
The dispersion adapted NLSE then is: $\frac{dA}{dz} = (B + N)A$. The nonlinear operator $N = \gamma|A|^2$ contains the nonlinear effects **[6]**. This NLSE will be solved using the Split-Step Fourier (SSF) algorithm **[7]**. From it, will appear a term exp $(i.q.z)$. In the stopband, q is purely imaginary, and the light is reflected, corresponding to a loss for the forward propagating pulse. This means the imaginary part of q must be positive, else the exponential term would diverge. The possibility to set the sign of q comes from the choice we have in $\beta_{BG} = \beta \pm q$. Not taking this into account results in the failure of the algorithm.

Nonlinear propagation in Bragg gratings

Previous experiment **[5]** about the nonlinear behavior of the grating close to the resonance was modeled via the nonlinear coupled mode equation (NLCME), which is robust in case where only the self-phase modulation and the cross-phase modulation exist, but is unsuited for BS-4WM. Using parameters like those of **[5]** we obtain a very good agreement between our simulation and the one from the reference. We observe on **Fig.2** a very good agreement between the two figures. The shape of the two output pulses is similar: a main peak that is intense then followed by a dip and a weak secondary peak. The time scales are also coherent for the output pulse duration and the delay between input and output pulse. The slight differences (main peak intensity, exact pulse duration) are probably due to the fact we use a standard fiber dispersion while the reference does not give the dispersion used to model their specific fiber. The very good agreement between our model and the NLCME, accompanied by the known ability of the NLSE to model 4WM, leads us to a high degree of confidence towards the algorithm predictions.

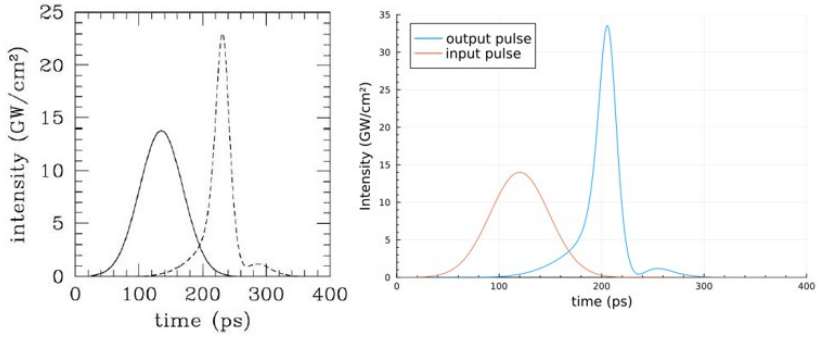

Fig.2: comparison between the numerical simulation using NLCME **[5]** and our model with the following parameters: (left) $\kappa \approx 10\text{cm}^{-1}$, L = 3.5cm, input pulse FWHM = 80ps, I = 14GW/cm^2, $\delta = 13.2\text{cm}^{-1}$ with a Bragg wavelength λ_B = 1053.175nm, (right) $\kappa \approx 10\text{cm}^{-1}$, L = 3.5cm, input pulse FWHM = 80ps, I = 14GW/cm^2, $\delta \approx 15.8\text{cm}^{-1}$ with a Bragg wavelength λ_B = 1053.293nm.

Algorithm prediction: BG enhance BS-4WM tunability

Using the dispersion adapted NLSE, we simulate the 4WM between 4 waves at 1550, 1308, 632 and 685 nm that takes place in a BG with a resonance close to 1548nm. We display in **Fig.3** the efficiency of the desired BS-4WM, defined as the ratio between the output idler power and the input signal power. The efficiency is shown as a function of the first pump wavelength ($\lambda_1$around 1550nm) and the second pump wavelength (λ_2 around 1308 nm) and compared to the efficiency when the grating is not present.

We observe as expected the similarity of the figures at the edges, as the grating influence becomes negligible. However, close to the grating resonance, we observe a different behavior. The first notable difference consists in the presence of a central gap where the efficiency plummet in the graph, corresponding to the grating bandgap. This is expected, as the grating is basically a reflector at resonance: For the pump close 1548 nm the energy is not transmitted which means the nonlinear processes are not fueled, and specifically, the BS-4WM cannot happen efficiently. On each side of this gap, we observe a drift in the maximum efficiency region. This drift is directly due to the influence of the grating on the wave vector of the first pump, hence on the phase matching condition. A lesser mismatch meaning a better efficiency. The asymmetry between the drifting direction (downward on the left side, upward on the right side) is easily understood with a reasoning on the influence of the grating on the dispersion. On one side, the grating can decrease the dispersion of the first pump β_1 (hence the phase mismatch $\Delta\beta = \beta_1 + \beta_i - \beta_2 - \beta_s$) which means higher efficiency area is moved toward one direction in regard of the vertical axe. This direction of the drift is set by the dispersion at λ_2: here increasing λ_2 decreases the wave vector $\beta_2$which itself increases $\Delta\beta$. So, starting from an almost phase-matched situation (without grating), the grating influence on the left side decreases the $\Delta\beta$ and is recovered for higher β_2 which means lower λ_2. The opposite is true on the other side of the gap, as the dispersion around λ_2 is locally monotonous.

From an initial situation without a good enough phase matching, we would have obtained a zone with enhanced efficiency on one side of the resonance, and one with decreased efficiency, on the other side. Another interesting aspect is the widening of the tunability. If we assume $\lambda_1$can be slightly changed (about a nm around the gap), the pump tunability is the amount of change on $\lambda_2$we can have while keeping a high efficiency (90% for example), with λ_{signal}fixed.

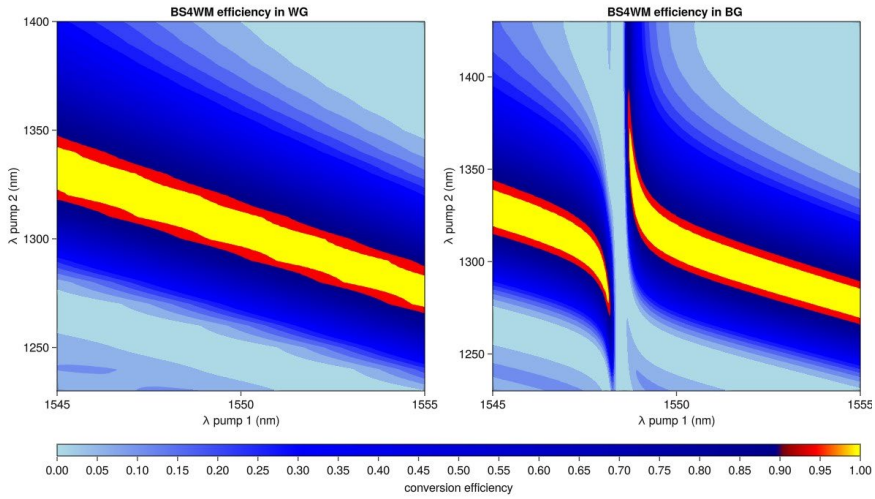

Fig.3: comparison between BS-4WM efficiency inside a waveguide (left) and inside an evanescent BG with same waveguide dimensions (right), as a function of the pumps wavelengths. The red-yellow part reveal pump configurations for which the BS-4WM conversion efficiency is between 0.9 and 1.

Without grating, the input pump tunability is about 40 nm, while with the grating it reaches 120 nm. This change is echoed on the span of λ_{idler} efficiently generated, which we called output tunability. The output tunability for the grating is slightly more than 30 nm (between 663 and 695 nm) while the output tunability without the grating is about 10 nm. If we look at the signal tunability on the other hand, i.e. λ_2 is now fixed, we have observed an input tunability about 3nm with the grating, and below 1nm without. This is translated into a 4.5nm output tunability with a grating, while it is 1nm without grating.

Conclusion

Encoding the Bragg grating influence into the dispersion operator enables the use of a dispersive-adapted NLSE to model how the grating interacts with the Four-Wave Mixing processes, and particularly the BS-4WM, in a fast and reasonable way. This also allows to consider all at once the undesired 4WM processes, which is not possible if we use a standard coupled-mode approach. Backed by a good agreement with previous numerical work backed experimentally, we use our algorithm to predict the Bragg grating to enhance the tunability of the BS-4WM by a factor of 3 compared to a case without grating. The model used here can be used for other 4WM processes in which the phase is relevant, providing the nonlinear phase to be reasonably low.

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