Symmetry-breaking in coupled microrings: a pathway to on-chip photonic Ising spin realization

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***Abstract*—Ising machines can efficiently encode and solve hard combinatorial optimization tasks as next-generation hardware accelerators. In this work, we develop an implementation of the coherent Ising machines (CIMs) on photonic integrated circuits (PICs). Coupled nonlinear microrings are used to realize Ising spins. We develop a theoretical model for an ideal on-chip photonic Ising spin. The imperfection in chip fabrication can lead to the inability of an Ising spin due to the destruction of symmetry-breaking conditions. We build a model to study the impact of the deviation in MMI splitters. As a next step, we will develop an optimization algorithm that can be applied to our programmable chips for compensating the imperfections.**

I. Introduction

The Ising model, known for its versatility in both theoretical and practical contexts, stands as a fundamental pillar in statistical mechanics. It provides invaluable insights into the behavior of interacting particles in various physical systems. It is depicted as an undirected graph, comprising a collection of binary Ising spin nodes ($x_i = \pm 1$) interconnected by Ising coupling interactions denoted as J_{ij}. The Ising Hamiltonian is mathematically expressed as $H = -\frac{1}{2}\sum_{i \neq j} J_{ij} x_i x_j$. The primary objective of solving an Ising problem is to determine spin configurations that minimize the Ising Hamiltonian, given a specific coupling interaction matrix. A generic Ising problem belongs to the category of NP-hard combinatorial optimization problems, indicating that it cannot be solved in polynomial time with respect to the problem size N (i.e. the number of nodes in the Ising model) and thus requires exponential computation time to solve on a classical computer. Due to the NP-hard nature, Ising problems have the potential to encode other NP-hard combinatorial optimization problems such as Maxcut problems and traveling salesman problems.

The operational principle of an Ising machine is to solve Ising problems by implementing an Ising model and minimizing its Ising Hamiltonian over time. Figure 1 provides an example of the workflow of using an Ising machine as a Maxcut solver.

In this work, we present the simulation of a novel designed Integrated Ising machine model that can be fabricated on photonic integrated circuits [1]. The main novelty lies in the consideration of a fifth-order nonlinearity to have more hyper-parameters and a large noise regime to facilitate exploration.

II. Integrated CIM model

As an important type of Ising machine, considerable attention has been paid to coherent Ising machines (CIMs) [2]–[4], which consist of a network of optical Ising nodes with bistable states of coherent light pulses (e.g., 0 and π phases). However, most of the CIMs are based on optical parametric oscillators (OPOs), requiring large experimental setups. As a compact Ising machine implementation, integrated CIMs have been proposed in recent work [1]. A schematic of an integrated CIM on a photonic integrated circuit can be seen in Figure 2.

The nonlinearity in the rate equation of each spin amplitude in the integrated CIM system allows bistable states above the threshold of input pumping power, which comes from the nonlinearities in the microring-based Ising unit, shown in Figure 3.

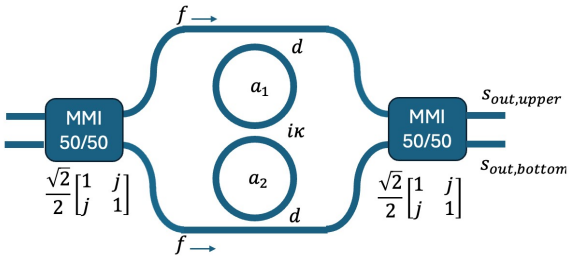

Fig. 3. An ideal on-chip Ising spin composed of two coupled nonlinear rings with two inputs equal in amplitude and phase. The two rings are designed to be identical.

The rate equations for the above coupled-ring system with Kerr nonlinearity (note that it can be extended to other nonlinearities):

$$\frac{da_1}{dt} = \left(i\left(\delta\omega + \frac{|a_1|^2}{U_0^e}\right) - 1\right) a_1 + i\tau\kappa a_2 + \tau df,$$
$$\frac{da_2}{dt} = \left(i\left(\delta\omega + \frac{|a_2|^2}{U_0^e}\right) - 1\right) a_2 + i\tau\kappa a_1 + \tau df,$$

where a_1 and a_2 are the amplitudes of modes in upper and lower rings, respectively; $\delta\omega$ is the initial detuning, U_0^e is the self-phase modulation (SPM) characteristic power, τ is the photon time, κ is the amplitude coupling coefficient between two rings, and f is the input signal to be coupled

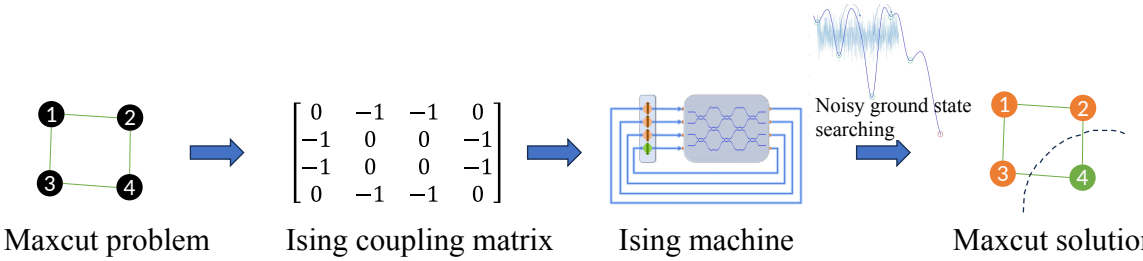

Fig. 1. Workflow of an Ising machine as a Maxcut solver: starting from a Maxcut problem defined by a certain undirected graph, the problem is then encoded into an Ising coupling matrix, which will be fed into an Ising machine with optimized hardware to this specific problem. By the noisy ground state search process, if the Maxcut problem is solvable, after an evolution time, the Maxcut solution will be finally returned as the ground-state Ising spin configuration.

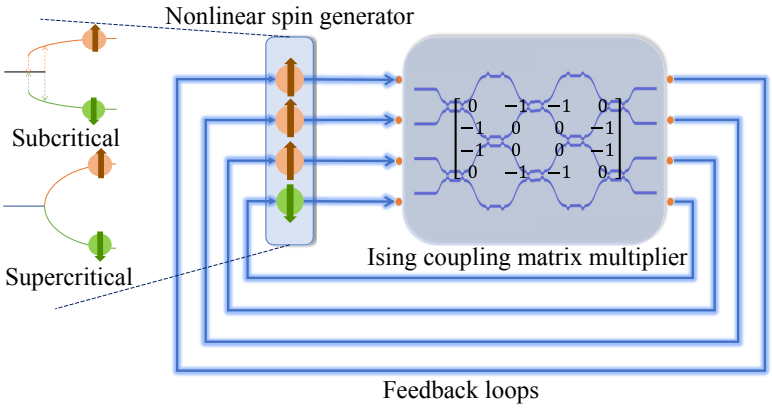

Fig. 2. Schematic of an integrated CIM on a photonics integrated circuit: three main building blocks are involved, including a nonlinear spin generator, an Ising coupling matrix multiplier, and feedback loops. The Ising machine can either be operated in subcritical pitchfork regime or supercritical pitchfork regime. The nonlinear spins and matrix multiplier can be realized by microring resonators and MZI mesh on a photonic integrated circuit.

into the rings. Note that the rate equations are invariant under the exchange of modes 1,2. However, under certain conditions, namely symmetry-breaking conditions, modes 1,2 can generate asymmetry steady-state solutions. By solving the steady states of $|a_1|^2$ and $|a_2|^2$ we can derive the symmetry-breaking condition:

$$\delta\omega - \tau\kappa < -\sqrt{3}.$$

III. Simulation Results

The circuit on our chip can be seen in Figure 4, consisting of an input matrix and output matrix with programmable phase shifters. In this section, simulation results of the circuit will be shown.

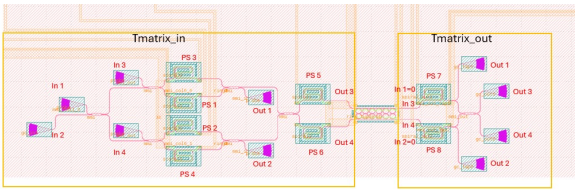

Fig. 4. Circuit on-chip for one Ising spin unit.

A. Symmetry-breaking in the ideal Ising Spin model

To achieve the desired Ising spin behavior, we require equal amplitude and phase of light at the input ports of the two rings. Operating under symmetry-breaking conditions, the Ising state can be effectively encoded in the bistable power states of the rings. This is necessary for the system's functionality, enabling the realization of an on-chip photonic Ising model. We take parameters under symmetry-breaking conditions and excite the circuit with sawtooth input signals, the symmetry-breaking result in the time domain can be seen in Figure 5.

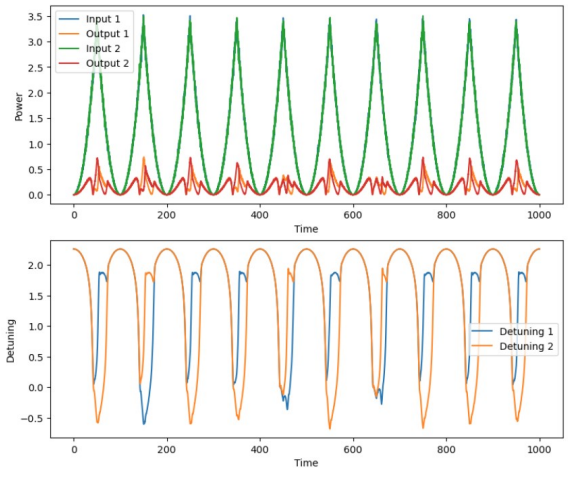

Fig. 5. Symmetry-breaking in coupled nonlinear rings with sawtooth inputs.

B. Fabrication imperfection and compensation

Real photonic circuits are usually fabricated with imperfections, e.g. deviations in a 50/50 MMI splitter. According to the study [1], the deviations in MMIs can significantly degrade the performance of the Ising machine. In this section, we investigate how the deviations in MMIs can be compensated in our circuit by tuning the phase shifters.

With a $\delta = 10$ deviation that exists for all the MMIs in the circuit, the amplitudes of light from output ports 3 and 4 as functions of phase shifters 3 and 4 are plotted in 3D in Figure 6. The equal amplitude points are desired for achieving symmetry-breaking, thus they are marked. The plot indicates that with an appropriate combination of ps_3 and ps_4 values, the deviation in MMIs can be compensated.

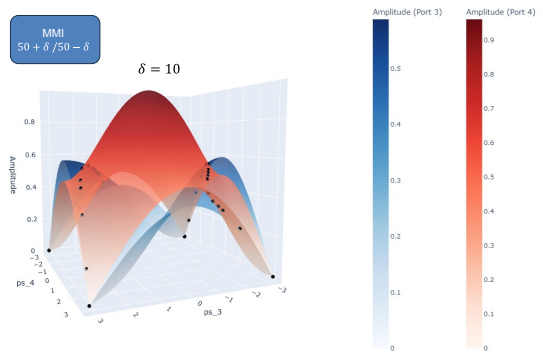

Fig. 6. The amplitudes of light from output ports 3 and 4 as functions of phase shifters 3 and 4 are plotted in 3D given imperfect MMIs.

IV. Conclusion

In this work, we demonstrated the realization of Ising spins on photonic integrated circuits (PICs) using coupled nonlinear microrings. We developed a theoretical model for an ideal on-chip photonic Ising spin and investigated the impact of fabrication imperfections on the system's symmetry-breaking conditions, which are essential for the Ising spin functionality. By modeling the effects of deviations in splitters, we gained insights into how these imperfections can hinder symmetry-breaking and thus the performance of the Ising spin. As a next step, we plan to design an optimization algorithm tailored for our programmable chips to compensate for these imperfections, enabling more robust and reliable on-chip photonic Ising models.

Acknowledgment

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References

[1] N. Tezak, T. Van Vaerenbergh, J. S. Pelc, G. J. Mendoza, D. Kielpinski, H. Mabuchi, and R. G. Beausoleil, "Integrated coherent ising machines based on self-phase modulation in microring resonators," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 26, no. 1, pp. 1–15, 2019.

[2] Z. Wang, A. Marandi, K. Wen, R. L. Byer, and Y. Yamamoto, "Coherent ising machine based on degenerate optical parametric oscillators," *Physical Review A*, vol. 88, no. 6, p. 063853, 2013.

[3] T. Inagaki, K. Inaba, R. Hamerly, K. Inoue, Y. Yamamoto, and H. Takesue, "Large-scale ising spin network based on degenerate optical parametric oscillators," *Nature Photonics*, vol. 10, no. 6, pp. 415–419, 2016.

[4] Y. Haribara, Y. Yamamoto, K.-i. Kawarabayashi, and S. Utsunomiya, "A coherent ising machine with quantum measurement and feedback control," *arXiv preprint arXiv:1501.07030*, 2015.