Examining the influence of hardware resolution on the efficiency of photonic Ising machines

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Ising machines offer a promising approach to solve complex optimization problems more efficiently than traditional digital computers. In these Ising machines, an optimization problem is encoded into a network of artificial spins, where the optimal solution is found by minimizing the system's energy through an iterative feedback process. Photonic implementations of such Ising machines are particularly promising due to their expected high operational speed and energy efficiency. Photonic Ising machines typically require optical modulators to implement the signals in these machines. However, modern day optical modulators have a relatively low modulation resolution. Based on numerical simulations, we investigate in this contribution how the limited bit-resolution of optical hardware has an influence on the performance of Ising machines. We find that the required resolution depends somewhat on the specific benchmark problem at hand, but in all investigated cases it remained below 8-bits. Surprisingly, we also find that the use of a 1bit-resolution modulator can improve the performance of the Ising machine by an order of magnitude across all considered benchmark problems.

Introduction

Many real-world combinatorial optimization problems cannot be efficiently solved using conventional digital computers, which are all based on the Von Neumann architecture. Examples of such problems can be found in the fields of logistics, finance, cryptography, and pharmaceuticals. For this type of problems, the number of possible (non-optimal) solutions grows very strongly with the number of variables in the problem.

In recent years, the Ising machine (IM) has emerged as a promising heuristic solver that could find the solution of this type of problems, commonly referred to as NP-hard problems, more efficiently. The reason behind this is that an IM is not built around the typical Von Neumann architecture but rather on the physical principle of energy-relaxation.

More specifically, the working principle of an IM is based on the mathematical concept that NP-hard problems can be mapped onto a network of binary variables, known as spins [1]. The couplings between these spins are selected such that the cost function of the original problem corresponds to the energy function of the IM, described by the equation:

$$H_{ising} = -\frac{1}{2}\sum_{ij} J_{ij}\,\sigma_i\sigma_j \qquad (1)$$

where the J_{ij} represents the coupling strength between the binary spins σ_i and σ_j. The optimal solution of a problem then correspond to the ground-state configuration of the network. Note that the spins are not stationary over time. At each time step, the spins are updated via a feedback loop that allows them to flip their sign. These spin flips can lower the network's energy, as defined in Eq. 1, enabling the system to find the ground-state energy, i.e., the solution.

There already exist multiple hardware implementations, which are mainly use electrical or optical components [1-3]. In our work, we will focus on optical implementations, as they offer several potential advantages, including higher bandwidth, lower latency, and greater energy efficiency. Furthermore, the use of spatial light modulators (SLMs) now enables the possibility of implementing large spin networks [3]. Remark that as modern display technology continues to evolve, IM will be able to leverage these advancements to further enhance the performance. In most of these hardware implementations, the spins in Eq. 1 are not strictly binary but analogue. Therefore, we work with a spin amplitude, x_i, which is afterwords mapped to a binary value using the simple function σ_i = sign(x_i) = ±1. Additionally, hardware implementations have limits on the maximum and minimum possible outputs, known as clipping effects. To mimic the behavior of hardware implementations in our numerical simulations, we use the hyperbolic tangent function in our dynamical evolution equation.

$$\frac{dx_i}{dt} = -x_i + \tanh(\alpha x_i + \beta \sum_j J_{ij} x_i) + \gamma \xi_i(t) \qquad (2)$$

where the $-x_i$ represent the linear loss in the network and $\xi_i(t)$ is gaussian white noise that is inherently present in an analogue hardware setup. α, β and γ are the hyperparameters that control the gain, feedback strength and noise strength respectively.

The argument of the tangent hyperbolic is referred to as the feedback signal that spin x_i receives each time step. In numerical simulations, this value is often represented by a 64bit floating point number. However, this level of precision is not possible in optical hardware, where feedback is sent to the analog spins via an optical modulator. These modulators typically have a relatively low resolution of around 8-bits. Therefore, it is crucial to determine the minimum resolution required for the optical modulator in an IM implementation with analogue spins.

In this work, we use numerical simulations to examine the influence of the bit resolution of the feedback signal on the efficiency of photonic IM implementations. Specifically, we digitize the feedback signal at different bit-resolutions and determine the bit-resolution at which the system behaves similarly to the floating-point simulations. From now on, we will refer to the undigitized signal and the digitized signal as the float-feedback and the bit-feedback, respectively.

Effect of bit-resolution

We use the Euler integration method to integrate the dynamical evolution equation in Eq. 2. This allows us to determine the temporal evolution of the analog spins, from which we can derive the associated energy evolution by inserting the sign of the analog amplitude into Eq. 1. Tracking the system's energy is crucial since the IM is a heuristic solver and does not guarantee that it will always find the ground-state energy of the problem. Consequently, the performance of an IM is often measured by its probability of finding the solution during its runtime. This metric is referred to as the transient success rate (TSR) and is defined as the ratio of runs in which the IM has found the solution to the total number of runs.

To incorporate the digitization that occurs at the modulator, we created divided the interval [-4, +4] into 2^n bins, where n is the applied bit-resolution. The range of this digitization interval is chosen to maximize the use of the entire output range, i.e. tanh(±4) ≈ ±1. Once the float feedback value is calculated, we employ a simple binary search

algorithm to round this value to the nearest bin within the digitization interval. The bit-feedback then becomes equal to the value of that nearest bin. We define the minimum required bit-resolution as the resolution at which the TSR becomes equal to the TSR of the float-feedback. Note that we have optimized the parameters α, β in Eq. 2 to maximize the TSR of the float-system. The noise strength γ=3 is kept constant throughout the simulations. We chose this noise level because it is high enough to introduce randomness into the system while remaining low enough such that it does not dominate the spin evolution.

Figure 1 illustrates how we determine the minimum required bit-resolution for a typical benchmark problem with 100 spin variables, i.e. the g05_100.02 MaxCut problem from the Biqmac library [4]. In this figure, the TSR is plotted as a function of the used bit-resolution. We calculated the TSR 100 times for each bit-resolution as well as for the float-feedback. From these data points, we determined the average TSR for both feedback methods (solid lines) and the associated standard deviation (shaded area). Note that the TSR of the float-feedback remains unchanged with different bit-resolutions, as it is independent of them.

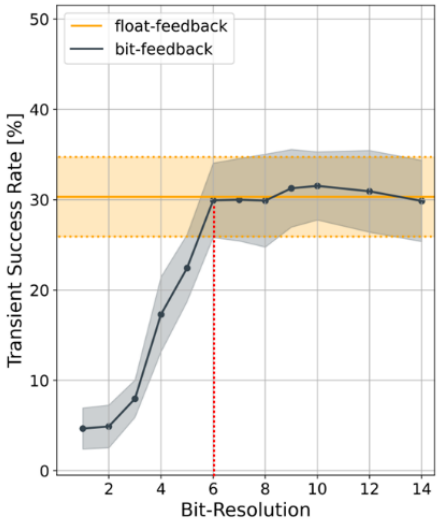

Figure 1: Transient success rate of the g05_100.2 optimization problem from the biqmac-benchmark library, without digitization of the feedback signal (indicated by "float-feedback") and with limited resolution of the feedback signal (indicated by "bit-feedback"), showing that for this problem, a resolution of 6 bits is sufficient to obtain similar results in both cases.

From this figure, we see clearly that at low bit-resolutions ($n < 6$), the TSR of the float-feedback is clearly higher than that of the bit-feedback. However, at 6-bit both TSR values start to overlap. We therefore conclude that for this specific problem, the minimum required bit-resolution is 6-bit. This is indicated by the red dotted line in the figure. We conducted these simulations on 10 additional benchmark problems, with 100spins. Then we repeated this study for different problem sizes containing 60, 80, 100, 300, 800, 1000 and 2000 spins. Our results show that the minimum required bit-resolution across all these problems is 8-bit or lower. This indicates that typical modern optical modulators are capable of accurately representing the feedback signal.

We also observe that the IM is still able to find the ground-state energy of the problem, even with only 1bit feedback resolution (see Figure 1), although its TSR is significantly lower compared to the float-feedback system. However, we argue that this comparison is not fair, as the parameters α and β were optimized specifically for the float-feedback system. To provide a more accurate comparison, we optimize the hyperparameters of both systems independently, resulting in different optimal values of α and β for the 1bit feedback and float-feedback systems.

Interestingly, across all ten tested benchmark problems, the IM with 1bit feedback reduced the time to find the solution by an order of magnitude compared to the IM that uses float-feedback. This suggests that, on average, the 1bit feedback IM requires significantly less time to find the solution to the optimization problem. Additionally, using lower-resolution feedback offers a fabrication advantage, as it typically results in a smaller footprint, improved energy efficiency, and reduced cost to implement the feedback in a physical system.

Summary

To summarize, this work discuss how the bit-resolution of optical modulators in hardware implementations of IMs impacts their performance. Additionally, we found that the IM can be accelerated by utilizing 1-bit feedback resolution. Both of these findings contribute to the further development of optical IMs.

References

[1] A. Lucas, Front. Phys. 2 (2104)

[2] T. Honjo, T. Sonobe, K. Inaba et al. , Sci. Adv. **7** (2021)

[2] F. Böhm, G. Verschaffelt and G. Van der Sande , Nat. Commun. **10** (2019)

[3] D. Pierangeli, G. Marcucci and C. Conti , Arxiv (2019)

[4] https://biqmac.aau.at/biqmaclib.html