An Investigation of Reflective Properties using Transformation Optics

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In this contribution, we use the geometrical formalism of transformation optics to investigate reflection at the interface between two media. We first highlight the difficulty of transformation optics when considering discontinuous coordinate transformations and, subsequently, we present reflective properties of discontinuously transformed media as a function of the coordinate stretching.

Transformation Optics

Transformation optics is a novel methodology in electromagnetism that uses the form invariance of Maxwell's equations under coordinate transformations to identify an equivalence between a medium with traditional electromagnetic material parameters, expressed on the background of a curved coordinate system, and a material with exotic material parameters, expressed in a traditional (Cartesian) coordinate system [1-2].

This equivalence has been used to design macroscopic metamaterials that allow for enhanced manipulation of electromagnetic waves based on geometrical deformations. The design procedure consists of three steps, as visualized in Fig. 1. One starts from light propagation in an empty space expressed in a Cartesian coordinate system [Fig. 1 (a)]. Subsequently, a coordinate transformation transforming the path of the light ray in a desired way, as shown in Fig. 1 (b), is applied. Using the equivalence relations of transformation optics, one then obtains the electromagnetic parameters of the specific (meta)material in which light propagates along this desired path [Fig. 1 (c)].

(a) (b) (c)

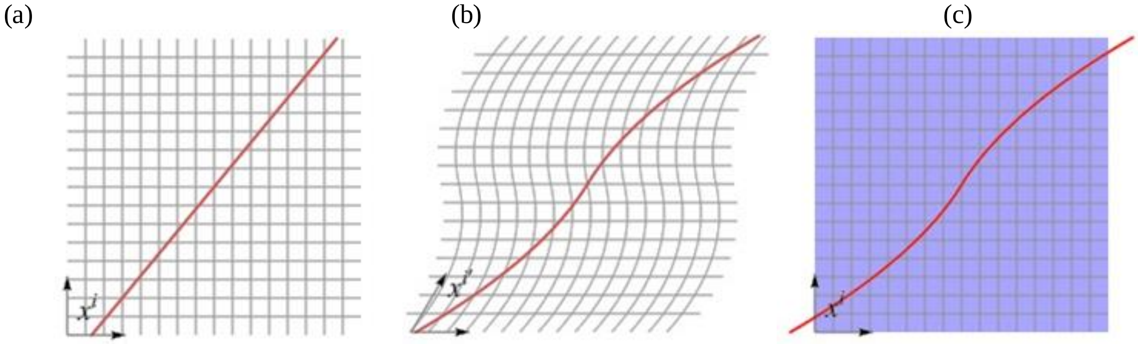

Fig. 4: *The design procedure of transformation optics: (a) the path of a light ray in empty space expressed in Cartesian coordinates, (b) is transformed under a coordinate transformation. (c) Using the equivalence relations, the effect of this coordinate transformation is implemented in a specific material.*

The Caveats of Transformation Optics at an Interface

So far, transformation optics has successfully been used to manipulate the propagation of electromagnetic beams through continuous media. However, at the interface between two media the formalism seems to break down. When we introduce a virtual interface in empty space expressed in Cartesian coordinates [Fig. 2 (a)] and transform only the vacuum region on the right-hand side of the interface [Fig. 2 (b)], a naïve interpretation of transformation optics would suggest that a light ray, propagating from the untransformed vacuum region into the material implementation of the transformed region on the right-hand side, will simply propagate through the material following the transformed path. In reality however, we found that in most cases reflections arise at the interface, as depicted in Fig. 2 (c).

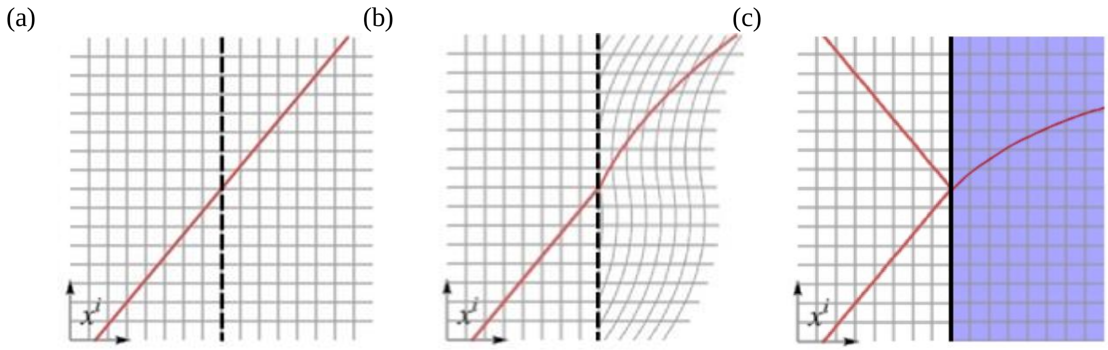

***Fig. 5**: The caveats of transformation optics at a discontinuous interface. (a-b) The naïve image of continuous light propagation into a discontinuously transformed space is not valid. (c) In reality, significant reflection and refraction will obscure the underlying geometry.*

Results and Numerical verifications

To obtain a better understanding of the physics at the interface between a vacuum region and a transformed region, we investigated the scattering at the interface from a first-principles approach, i.e., using the full Maxwell equations in combination with the proper boundary conditions [3]. For the interface between a vacuum region and a material implementing an isotropic stretching of the coordinate lines, we found analytical expressions for the reflected and refracted beams. Using this analytical model, different reflective regimes can be distinguished, depending on the stretching parameter a of the coordinate transformation and on the angle of incidence θ_{in}. The derived expressions for the reflection and transmission coefficients were verified numerically by full-wave simulations using a finite-elements solver, both for TE and TM polarization. Figure 3 shows the analytical model (blue lines) for the magnitude (a) and the phase angle (b) of the reflection coefficient for TE polarized light, together with the numerical verifications (red circles), for an isotropic stretching with stretching parameter a.

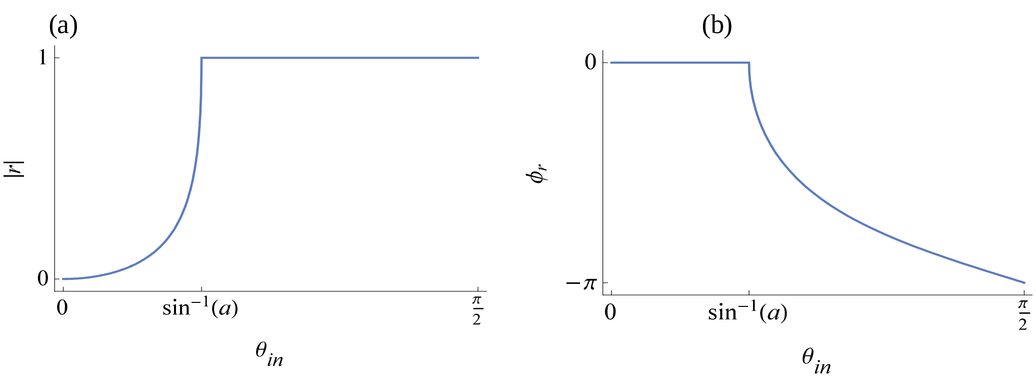

***Fig. 6**: Magnitude (a) and phase angle (b) of the reflection coefficient r as a function of the angle of incidence θ_{in} for a TE-polarized wave, incident on an interface between a vacuum region and a material implementation of an isotropic stretching with stretching parameter a*

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